McMaster University Math 1XX3 Winter 2013
Midterm 2 — Practice Version
Duration: 60 minutes

Instructor: Dr. D. Haskell

Name: ____

Student ID Number: _______________

This test paper is printed on both sides of the page. There are 6 question on 5 pages, with a blank page at the end for rough work. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator.

Instructions

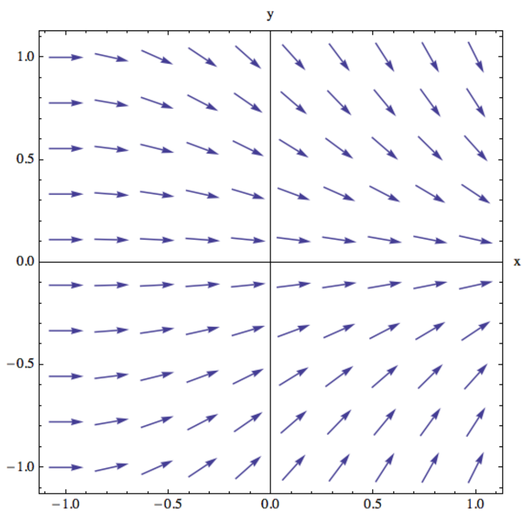
(1) Only the standard McMaster calculator is allowed.
(2) All answers must be written in the space following the question. If you need more space, use the blank page at the end of the exam, and indicate clearly where to find the answer.
(3) Additional scratch paper is available for rough work; ask the invigilator.

This PRACTICE version of the midterm is intended to give you an idea of the format, approximate length and approximate difficulty of the actual midterm. There is no guarantee as to the actual length and difficulty of the actual exam. In particular, the actual midterm will NOT be "just the same with the numbers changed".

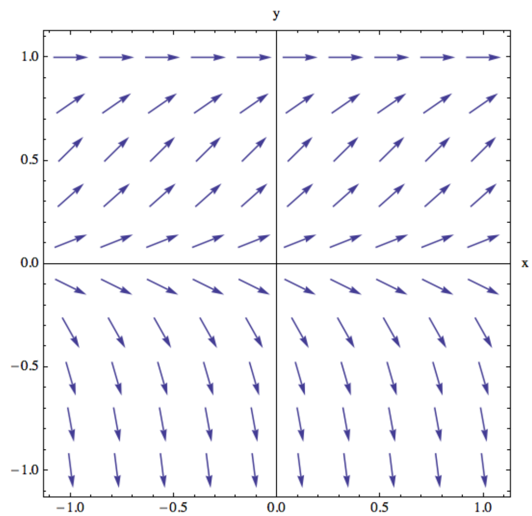
Problem	Points
1 [10]	
2 [5]	
3 [5]	
4 [5]	
5 [5]	
6 [5]	
Total [35]	

1) [10 points]

a) There are two slope fields and three differential equations given. Match the slope field with the correct differential equation.



A. _____



B. _____

1. $\dfrac{dy}{dx} = y - x$ 2. $\dfrac{dy}{dx} = 4y(1-y)$ 3. $\dfrac{dy}{dx} = -y(1+x)$

b) Let $f(x) = (x^2+4)^{1/2}$. Express the length of the path along the curve $y = f(x)$ from $x = 0$ to $x = 3$ as an integral. DO NOT solve the integral.

c) Explain briefly why the differential equation $\frac{dI}{dt} = kI(t)$ is not a good model for the spread of a disease in a population. ($I(t)$ is the number of people infected at time t.)

d) Describe the path of a particle (both shape of curve and how the curve is traversed) which moves so that its position in the plane at any time t is given by the parametric equations $x(t) = \sin(t)$, $y(t) = 2\sin(t)$.

e) Find polar coordinates for the point with cartesian coordinates $(1, -2)$.

2) [5 points] Solve the linear differential equation $x\dfrac{dy}{dx} - y = x\ln(x)$ with initial condition $y(1) = 2$.

3) [5 points] A curve passes through the point $(4, 2)$ and has the property that the slope at every point $P(x, y)$ on the curve is 3 times the product of its x coordinate and its y coordinate. Find the equation of the curve.

4) [5 points] Find the surface area of the body obtained by rotating the graph of $y = x^3$ around the x-axis from $x = 0$ to $x = 3$.

5) [5 points] Calculate a table of values and then sketch the curve given in polar coordinates by the equation $r = \sin(2\theta)$.

6) [5 points] Consider the curve given in parametric form by the equations

$$\begin{aligned} x(t) &= t^3 - 3t + 1 \\ y(t) &= t^3 - 3t^2 + 1 \end{aligned}$$

Find the values of the parameter at which the tangent line to the curve is horizontal and the values of the parameter at which the tangent line to the curve is vertical. Hence find the points on the curve at which the tangent line is horizontal and the points on the curve at which it is vertical.