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Weak and strong reversibility of causal processes.

Stochastic causality (*Stanford, CA*, 1998; *Bertinoro*, 1999), 203–220, *CSLI Lecture Notes*, 131, *CSLI Publ.*, *Stanford, CA*, 2001.
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The author proposes to alleviate the tension between the well-known invariance under time reversal for the basic equations of physics and the empirical distinction between past and future observed in everyday life, by introducing the concepts of "weak" and "strong" reversibility for causal processes.

A process is said to be weakly reversible if it is mapped under time reversal into a process of the same class. Examples of such processes are Newtonian as well as relativistic particle mechanics, for which the author provides a heavily technical proof in the appendix (a summary of two of his papers on the subject from the 1950's). The unitary evolution in quantum mechanics (given by the Schrödinger equation) is also given as an example of a process with weak reversibility, while the quantum mechanical measurement process is a counterexample.

Next the paper defines a process as strongly reversible if we are unable to distinguish whether it is running forward or backward. The rest of the paper is dedicated to a discussion of strongly reversible processes.

An example considered in greater detail is that of a Markov chain X with a finite number of states. Before stating the main result the author provides several definitions pertaining to such stochastic processes (such as those of irreducible, aperiodic, stationary and ergodic chains), some more accurate than others. For instance, his definition of an ergodic chain as being one with a unique asymptotic stationary distribution is reversed. Ergodicity is more properly defined by the property that all (non-negligible) sets in the state space are eventually going to be visited by the process. For finite Markov chains this is equivalent to his previous definition of irreducibility (any two states can be connected in a finite number of steps) and then it is a highly nontrivial result to show that this implies the existence of a unique asymptotic stationary distribution.

In any event, the author rephrases the notion of strong reversibility in terms of what is known in physics as "detailed balance", namely the property that $\pi_i p_{ij} = \pi_j p_{ji}$, for some distribution π (which is then easily proved to be stationary), where p_{ij} is the transition probability (matrix) from the state i to the state j.

The main result of the paper is the proof that this reduces to the previous wordy definition of strong reversibility, namely that one cannot distinguish past from future by observing such a process, since $(X_0, X_1, \dots, X_n)$ has the same joint distribution as $(X_n, X_{n-1}, \dots, X_0)$.

The result is stated for continuous time Markov processes with finite state space, but no proof is offered. However, better than a proof, the author gives the intriguing example of the Ehrenfest model, consisting of two containers of ideal gas molecules with $X(t)$ representing the number of particles in one of them. For transition probabilities proportional to the existing number of particles in each container, he explicitly shows that such a process is strongly reversible. The striking feature is that, despite strong reversibility, this system exhibits a well-defined arrow of time, even at equilibrium, due to an always increasing entropy. At this point we are left in the dark as to why these two features are "obviously" not contradictory, and are referred to the work of Mark Kac for a detailed analysis.

The paper concludes with a rather trivial analysis of two deterministic systems, namely the undriven harmonic oscillator with and without damping. The latter is shown to be strongly reversible, while the former, as is the case for any dissipative system, is not.

{For the entire collection see MR1944377 (2003f:00024)}

Reviewed by *M. R. Grasselli*