Online appendix for ‘Testing a goodwin model with general capital accumulation rate’

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In the main text of Grasselli and Maheshwari (2018) we provide econometric estimates for a modified Goodwin model using data for 10 OECD countries. The purpose of this appendix is to provide the corresponding estimates for three related models: the original Goodwin model (Goodwin, 1967), the Desai extension incorporating inflation and a variable capital-to-output ratio (Desai, 1973), and the van der Ploeg extension using a more general production function (van der Ploeg, 1985).

Keywords: Goodwin model, endogenous cycles, parameter estimation, employment rate, income shares.

JEL Classification Numbers: C13, E11, E32.

1 The original Goodwin model

This corresponds to setting $k = 1$ in equation (5) in Grasselli and Maheshwari (2018). Using the estimates for the remaining parameters obtained in Grasselli and Maheshwari (2018) leads to the results reported in Table 1. Since the investment-to-profit ratio does not affect the estimates for equilibrium employment rate, we see that the only differences between the original Goodwin model and the modified model analyzed in Grasselli and Maheshwari

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(2018) are the estimated equilibrium wage shares ω_G and period T_G. Because the estimates $\widehat{k}$ obtained in Grasselli and Maheshwari (2018) are smaller than one for all countries, we observe that the estimates of ω_G for the original Goodwin model are systematically *larger* than the corresponding values for the modified model, whereas the opposite holds for the estimates of T_G.

[Insert Table 1 here]

As we can see in Figure 1, the equilibrium wage share ω_G for the original Goodwin model fall outside the range of observed wage shares for all countries. Given the unrealistic assumptions of the model, this should not be surprising. This motivated us to investigate more generalized versions of the Goodwin model in the next Sections, where the assumptions of constant capital-to-output ratio, perfect complementarity of labour and capital and real bargaining are relaxed.

2 The Desai model

The first extension we consider consists of a model introduced in Desai (1973) to incorporate inflation and a variable capital-to-output ratio into the Goodwin model. In this section, we will first discuss the theoretical setup of the model as proposed in Desai (1973) and the corresponding econometric setup as discussed in Desai (1984), followed by our own estimation results.

2.1 Model Setup

The model attempts to address two important shortcomings of the Goodwin model: wage bargain in terms of real wages and a constant capital-to-output ratio. For the first, Desai (1973) incorporates an expected inflation rate $\left(\frac{\dot{p}}{p}\right)^e$ in the wage bargaining equation (1), where $m = p\mathrm{w}$ is the nominal wage rate obtained by multiplying the real wage rate w by an appropriate price index p. The adjustment between expected and actual inflation takes place according to equation (2), with the speed of adjustment governed by the constant ζ, whereas the price dynamics itself is given by equation (3), which says that prices adjust to an equilibrium given by labour costs m/a times a constant markup factor π, with the speed of adjustment governed by the constant χ.

$$\frac{\dot{m}}{m} = \gamma + \rho\lambda + \eta(\frac{\dot{p}}{p})^e \tag{1}$$

$$\frac{d}{dt}\left(\frac{\dot{p}}{p}\right)^e = \zeta\left\{\frac{\dot{p}}{p} - \left(\frac{\dot{p}}{p}\right)^e\right\} \tag{2}$$

$$\frac{\dot{p}}{p} = \chi(\log\omega + \log\pi) \tag{3}$$

Next the assumption of a constant capital-to-output ratio is replaced by assuming a dependence on the employment rate of the form

$$\nu = \nu^*\lambda^{-\kappa}, \tag{4}$$

for a constant κ. Using these changes, we can derive the state equations as[1]:

$$\frac{d}{dt}\left(\frac{\dot{\omega}}{\omega}\right) = (\gamma - \alpha)\zeta + \rho\zeta\lambda + \rho\dot{\lambda} + (\eta - 1)\zeta\chi(\log\omega + \log\pi) - (\chi + \zeta)\frac{\dot{\omega}}{\omega} \tag{5}$$

$$\frac{\dot{\lambda}}{\lambda} = -\frac{\alpha + \beta + \delta}{1 - \kappa} + \frac{(1 - \omega)\lambda^\kappa}{\nu^*(1 - \kappa)} \tag{6}$$

Now, following Desai (1984) define

$$x_1 = \log\lambda \tag{7}$$

$$y_1 = \log\omega \tag{8}$$

$$y_2 = \dot{y}_1 \tag{9}$$

[1] It can be seen that the coefficients of some of the terms in (5) are not the same as given by Desai (1984). This will impact the equilibrium estimates of wage share and employment rate.

so that the second-order system (5)-(6) can be written as the first-order system

$$\dot{x}_1 = -\frac{\alpha+\beta+\delta}{1-\kappa} + \frac{1}{\nu^*(1-\kappa)}(1-e^{y_1})e^{\kappa x_1} \tag{10}$$

$$\dot{y}_1 = y_2 \tag{11}$$

$$\dot{y}_2 = -(\alpha-\gamma)\zeta + (\eta-1)\zeta\chi\log\pi + (\eta-1)\zeta\chi y_1 - (\chi+\zeta)y_2 + (\rho\zeta+\rho\dot{x}_1)e^{x_1} \tag{12}$$

At equilibrium, $\dot{x}_1 = \dot{y}_1 = \dot{y}_2 = 0$ and (10) gives

$$\omega = 1-(\alpha+\beta+\delta)\nu^*\lambda^{-\kappa}, \tag{13}$$

whereas (12) gives

$$\lambda = \frac{(-\gamma+\alpha)+(1-\eta)\chi\log\pi}{\rho} + \frac{(1-\eta)\chi}{\rho}\log\omega. \tag{14}$$

As expected, these functions reduce to the Goodwin equilibrium values when $\kappa = 0$ and $\eta = 1$. In Figure 2, we illustrate the solution of equations (13) and (14). The intersection of the two equations give us the required equilibrium estimates ω_D and λ_D. Although there are two equilibria, we will only consider the higher one, as the other is unrealistically low.

[Insert Figure 2 here]

2.2 Econometric Setup

To estimate the modified Phillips curve given by equations (1) and (2), Desai (1984) proposes the following discretization:

$$\Delta\log(m_t) = \gamma + \rho\lambda_t + \eta\Delta\log(p_t^e) + \epsilon_{1t} \tag{15}$$

$$\Delta\log(p_t^e) = \zeta\Delta\log(p_t) + (1-\zeta)\Delta\log(p_{t-1}^e) + \epsilon_{2t} \tag{16}$$

As can be seen from these equations, $\zeta = 1$ corresponds to the case of "perfect expectations", whereby changes in $\log(p_t^e)$ exactly match changes in $\log(p_t)$ (apart from a random error), whereas $\eta = 1$ corresponds to the "absence of money illusion", whereby changes in expected inflation are incorporated one-to-one into changes in nominal wages. In other words, the restriction $\zeta = \eta = 1$ reduces the model to the original Goodwin model of the previous section, and will be subjected to econometric tests in what follows.

Since expected inflation is not measurable, we need to eliminate it from the estimation equation. After some rearrangements, equations (15)-(16) reduce to

$$\begin{aligned}\Delta^2 \log(m_t) =& \zeta\gamma + \zeta\rho\lambda_{t-1} + \rho\Delta\lambda_t + \eta\zeta\Delta\log(p_t) - \zeta\Delta\log(m_{t-1}) \\ &+ \zeta\epsilon_{1t-1} + \Delta\epsilon_{1t} + \eta\epsilon_{2t}\end{aligned} \tag{17}$$

We can easily estimate this using nonlinear least squares. To get an estimate of π and χ, the two variables in the markup equation (3), we perform linear regression on the following discretized equation:

$$\Delta\log(p_t) = \chi\log\pi + \chi(\log(m/a)_t - \log(p_{t-1})) + \epsilon_{3t}. \tag{18}$$

Finally, for variable capital-to-output ratio (4), we estimate the following equation:

$$\Delta\log\nu_t = -\kappa\Delta\log\lambda_t + \epsilon_{4t}. \tag{19}$$

Notice that we use the differenced version of the log-transformed equation (4) to have stationary variables for regression, since both capital output ratio ν and employment rate λ are non-stationary. Estimating the parameters in equations (17)-(19) provide all the estimates necessary to compute the equilibrium values in (13)-(14), except ν^*.

2.3 Estimation results

We first estimate the expectation-augmented Phillips curve in (17) by using nonlinear least squares. The results are presented in Table 7. We observe that employment rate has the least impact on the rate of change of nominal wages in the UK and the most impact in Norway, with coefficients being 0.142 and 1.32 respectively. Moreover, the coefficient is significant for all the countries. Also, the hypothesis $\eta = 1$ can be rejected only for Norway at 5% level and the hypothesis $\zeta = 1$ can be rejected for 5 of the 10 countries examined, where ζ is strictly less than 1, but cannot be rejected for the rest. The joint hypothesis that $\eta = \zeta = 1$ cannot be rejected for Denmark, France, Italy, UK and Germany. Thus, for these countries, the estimates for γ and ρ are similar to what we derived in the previous section for linear Phillips curve and the assumption of real wage bargaining in basic Goodwin model is not that stringent after all. For the remaining countries, however, the impact of inflation should not be ignored.

Our results contrast with Desai (1984), where the same model is estimated for the UK for a number of sub-periods from 1855-1965: Desai finds that in the cases when the hypothesis $\zeta = 1$ is rejected, the hypothesis $\eta = 1$ is not rejected, and vice-versa, with the joint hypothesis $\eta = \zeta = 1$ being always rejected. Similarly, using data for OECD countries from 1954 to 1994, Harvie (2000) finds that the joint hypothesis $\eta = \zeta = 1$ is rejected for all 10 countries in his sample, leading him to conclude that the absence of expected inflation in the bargaining equation in the original Goodwin model is not justified.

Secondly, we estimate the parameters in the markup equation (18). Table 8 shows that the markup factor is around 1.6 for all the countries except Norway, where it is 2.2. Also, the adjustment term χ is significantly less than 1 for all the countries.

Next we estimate of the parameter κ in the variable capital-to-output equation (19), which also tests the assumption of constant capital output ratio in the Goodwin model, corresponding to $\kappa = 0$. We find that this parameter estimate varies between 0.4 to 1.8 and it remains significant for 7 of the 10 countries examined, the exceptions being Australia, Italy and Norway. This indicates that the strong assumption of a constant capital-to-output ratio made in the Goodwin model should be abandoned in most countries. This is consistent with the result reported in Harvie (2000), where it is found that the hypothesis $\kappa = 0$ is rejected for all countries except Italy and the UK.

Table 2 summarizes the results for the Desai model. The estimates for the parameters γ, ρ, η and ζ appearing in (1) and (2) are taken from Table 7, whereas the estimates for the parameters χ and π appearing in (3) are taken from Table 8. Using the estimates for α, β and δ found in the previous section (see Table 1), we can now find the equilibrium estimates given by intersection of equations (13) and (14). Since we could not find the estimate of ν^*, we will consider different values between 2.5 to 3 to understand its impact, as real data shows that the capital-to-output ratio lies around 3 for most countries during the period of study. The last two columns in Table 2 shows the values for ω_D and λ_D obtained as the intersection of (13) and (14) using $\nu^* = 3$, as this is the most favourable estimate when compared to the corresponding empirical averages.

[Insert Table 2 here]

We depict the estimates of equilibrium wage share and employment rate for this model along with observed data in Figure 1. We observe no noticeable improvement with respect to the original Goodwin model. Whereas the average relative error between estimated and observed wage share is slightly reduced from 13.6% in the Goodwin model to 12.6% in the Desai model, the corresponding error for the employment rate, although very small for both models, is actually increased from 0.52% in the Goodwin model to 0.87% in the Desai model. Thus although the model gave quality insight into the impact of inflation, equilibrium estimates continue to be unsatisfactory and we need to relax other assumptions to get meaningful estimates of equilibrium wage share.

3 The van der Ploeg Model

3.1 Model Setup

The model in van der Ploeg (1985) incorporates substitution between labor and capital into the Goodwin model by using the CES production function

$$Y(t) = A[\mu K(t)^{-\theta} + (1-\mu)L_e(t)^{-\theta}]^{-\frac{1}{\theta}} \tag{20}$$

where $A > 0$ and $0 < \mu < 1$ are constants and $L_e(t) = a^* e^{\alpha t} L(t)$, for constants a^* and α. The elasticity of substitution is defined as $\sigma = \frac{1}{1+\theta}$, and we observe that as $\theta \to 0$ we have that $\sigma \to 1$ and the production function converges to the Cobb-Douglas function $Y(t) = AK^\mu L_e^{1-\mu}$, whereas as $\theta \to \infty$ we have that $\sigma \to 0$ and the production function converges to the Leontieff function $Y(t) = \min(AK(t), AL_e(t))$. In other words, we recover the Goodwin model with a Leontief production function, constant capital-to-output ratio ν^*, and productivity $a(t) = a_0 e^{\alpha t}$ by setting $\theta = \infty$, $A = 1/\nu^*$, and $a^* = a_0 \nu^*$.

Using the hypothesis that $\frac{\partial Y}{\partial L} = \mathrm{w}$, van der Ploeg finds that the optimal capital-to-output ratio and productivity are given by

$$\nu(t) = \frac{K(t)}{Y(t)} = \frac{1}{A}\left(\frac{1-\omega(t)}{\mu}\right)^{-\frac{1}{\theta}} \tag{21}$$

$$a(t) = \frac{Y(t)}{L(t)} = a^* A e^{\alpha t}\left(\frac{\omega(t)}{1-\mu}\right)^{\frac{1}{\theta}}. \tag{22}$$

The model proposed in van der Ploeg (1985) still assumes wage bargaining in real terms. However, as we have seen in the previous section, inflation can have substantial impact on the bargaining behaviour of workers. Accordingly, we present below a variant of the van der Ploeg model incorporating inflation in the Phillips curve.

Version 1 - Real Philips curve

This is the original model in van der Ploeg (1985), with wage bargaining of the form:

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \varrho\frac{\dot{a}}{a} = \gamma + \rho\lambda + \varrho\frac{\dot{a}}{a}, \tag{23}$$

for constants γ, ρ, ϱ. This leads to the following differential equations for wage share and employment rate:

$$\frac{\dot{\omega}}{\omega} = \frac{\gamma + \rho\lambda - (1-\varrho)\alpha}{1 + \frac{1-\varrho}{\theta}} \tag{24}$$

$$\frac{\dot{\lambda}}{\lambda} = A\mu^{-1/\theta}(1-\omega)^{\frac{1+\theta}{\theta}} - \frac{\gamma + \rho\lambda - (1-\varrho)\alpha}{(1-\omega)(\theta + 1 - \varrho)} - (\alpha + \beta + \delta). \tag{25}$$

The equilibrium point for the system above is given by

$$\overline{\lambda} = \frac{(1-\varrho)\alpha - \gamma}{\rho} \tag{26}$$

$$\overline{\omega} = 1 - \left(\frac{\alpha + \beta + \delta}{A}\right)^{1-\sigma} \mu^{\sigma}, \tag{27}$$

As expected, this reduces to the equilibrium values for the Goodwin model when $\varrho = 0$, $\theta = \infty$ (or equivalently $\sigma = 0$) and $A = 1/\nu^*$ for a constant capital-to-output ration ν^*. Unlike the Goodwin model, the equilibrium above is a stable sink provided $0 < \sigma < 1$ and $\varrho\sigma < 1$. In particular, this shows that the Goodwin model is structurally unstable, since a small perturbation away from $\sigma = 0$ turns the closed cycles into spiral orbits converging a sink (see Figure 3).

[Insert Figure 3 here]

Version 2 - Nominal Philips curve

Here we modify the model in van der Ploeg (1985) with a nominal wage bargaining of the form (28) and inflation dynamics given by equation (29):

$$\frac{\dot{m}}{m} = \gamma + \rho\lambda + \eta\frac{\dot{p}}{p} \tag{28}$$

$$\frac{\dot{p}}{p} = \chi(\log\omega + \log\pi), \tag{29}$$

where, as before, π is a constant mark-up factor. We then get the following system of differential equations for wage share and employment rate:

$$\frac{\dot{\omega}}{\omega} = \frac{\gamma + \rho\lambda - \alpha - (1-\eta)\chi\log(\omega\pi)}{1+\frac{1}{\theta}} \tag{30}$$

$$\frac{\dot{\lambda}}{\lambda} = A\mu^{-1/\theta}(1-\omega)^{\frac{1+\theta}{\theta}} - \frac{1}{\theta(1-\omega)}\frac{\dot{\omega}}{\omega} - (\alpha+\beta+\delta) \tag{31}$$

with the corresponding equilibrium values:

$$\overline{\omega} = 1 - \left(\frac{\alpha+\beta+\delta}{A}\right)^{1-\sigma}\mu^{\sigma} \tag{32}$$

$$\overline{\lambda} = \frac{\alpha - \gamma + (1-\eta)\chi\log(\omega\pi)}{\rho} \tag{33}$$

As expected, these reduce to the Goodwin equilibrium values when $\theta = \infty$ (or equivalently $\sigma = 0$), $\eta = 1$, and $A = 1/\nu^*$ for a constant capital-to-output ratio ν^*. Unlike the Goodwin model, it is easy to see that the equilibrium above is a stable sink provided $\eta < 1$ and $\sigma < 1$.

3.2 Econometric Setup

There are numerous econometric techniques to estimate the CES production function, such as ordinary least squares (OLS) estimates using the first order conditions, linear approximation as proposed in Kmenta (1967), optimization algorithms for nonlinear functions, instrumental variables and the systems approach. Based on the detailed discussion of the problems and advantages of each of these approaches presented in León-Ledesma et al. (2010), we chose to use the systems approach on lines of seemingly unrelated regression.

Following León-Ledesma et al. (2010), we normalize the variables in the production function to make the interpretation of the parameters meaningful and consistent with the basic properties of the CES production function in the context of the growth theory. For this, observe first that, under the profit-maximization assumption adopted in the van der Ploeg model, the return on capital defined takes the following form:

$$r(t) = \frac{\partial Y}{\partial K} = \frac{\mu}{A^{\theta}}\left(\frac{Y(t)}{K(t)}\right)^{\theta+1}. \tag{34}$$

Next, let (r_0, w_0) be the arithmetic mean of the variables $(r(t), \mathrm{w}(t))$ and let (Y_0, K_0, L_0) be the geometric mean of the growing variables $(Y(t), K(t), L(t))$. We can then rewrite the CES production function in equation (20) as

$$Y(t) = \xi Y_0 \left[\mu_0 \left(\frac{K_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu_0) \left(e^{\alpha t} \frac{L_t}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{35}$$

where the new constants are related to the old constants through

$$\mu_0 = \frac{r_0 K_0}{r_0 K_0 + \mathrm{w}_0 L_0} \tag{36}$$

$$A = \xi Y_0 \left[\frac{r_0 K_0^{1/\sigma} + \mathrm{w}_0 L_0^{1/\sigma}}{r_0 K_0 + \mathrm{w}_0 L_0} \right]^{\frac{\sigma}{\sigma-1}} \tag{37}$$

$$\mu = \frac{r_0 K_0^{1/\sigma}}{r_0 K_0^{1/\sigma} + \mathrm{w}_0 L_0^{1/\sigma}}. \tag{38}$$

We then estimate the following system of three equations, consisting of the production function itself and its two first-order partial derivatives:

$$\log\left(\frac{Y(t)}{Y_0}\right) = \log \xi + \frac{\sigma}{\sigma-1} \log \left[\mu_0 \left(\frac{K(t)}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu_0) \left(e^{\alpha t} \frac{L(t)}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right] \tag{39}$$

$$\log r(t) = \log\left(\mu_0 \frac{Y_0}{K_0}\right) + \frac{1}{\sigma} \log\left(\frac{Y(t)/Y_0}{K(t)/K_0}\right) + \frac{\sigma-1}{\sigma} \log \xi \tag{40}$$

$$\log \mathrm{w}(t) = \log\left((1-\mu_0) \frac{Y_0}{L_0}\right) + \frac{1}{\sigma} \log\left(\frac{Y(t)/Y_0}{L(t)/L_0}\right) + \frac{\sigma-1}{\sigma} [\log \xi + \alpha(t-t_0)] \tag{41}$$

For estimating the Phillips curve specified in equation (23) in Model 1, we use the autoregressive distributed lag (ARDL) bounds-testing approach, since we know that the productivity growth rate is stationary for all the countries, wheres the other two variables (wage growth rate and employment rate) have a mix of stationarity and non-stationarity depending on the country.

On the other hand, for estimating the Phillips curve specified in equation (28) in Model 2, we use Johansen's co-integration test (see Johansen, 1988, 1991) whenever all three variables (wage growth rate, employment rate and inflation) in the nominal Phillips curve are non-stationary, and the autoregressive distributed lag (ARDL) bounds-testing approach otherwise. Johansen's approach uses a Vector Error-Correction Model (VECM) to test the hypothesis for co-integration. In the current setting, if we define the vector $y_t = [\frac{\dot{m}}{m}\ \frac{\dot{p}}{p}\ \lambda]$, then the VECM is

$$\Delta y_t = P(y_{t-1} + C_0) + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + ... + B_q \Delta y_{t-q} + C_1 + \epsilon_t \qquad (42)$$

where P and B_i, $i = 1, \ldots, q$ are 3×3 matrices. Now, P can be thought of as a product AB'; where, A and B are $3 \times n$ matrices and $n < 3$ (the rank of the matrix P) refers to the number of co-integrating relationships. The coefficients of A determine the size of the effects of the k error correction terms in the 3 equations of the Vector Error-Correction Model. To test for the number of co-integrating relationships given by n, we use trace tests (see Johansen, 1988, 1991), where the null hypothesis is $H_0 : n \leq n^*$ for some hypothesized value of $n^* \in \{0, 1, 2\}$. If $H_0 : n \leq 0$ is accepted, then $P = 0$, there is no co-integration, and the VECM reduces to a Vector Autoregressive (VAR) model in differences. Otherwise we test $H_0 : n \leq 1$ next, which if accepted implies that we have exactly one co-integrating vector, and similarly for $H_0 : n \leq 2$. Finally, if $H_0 : n \leq 2$ is also rejected, then P has full rank and Py_{t-1} spans the same vector space as y_{t-1}. Therefore, at least one element of y_{t-1} is $I(1)$ which contradicts the assumption in the VECM equation (42).

3.3 Estimation Results

Our estimation methodology for the production function is very similar to León-Ledesma et al. (2010) and the results are presented in Table 9. We used the `nlsystem` function within the RATS software package to estimate the nonlinear system of equations (inputs to the optimization algorithm included 1 million sub iterations, 100 thousand iterations and the convergence criterion of 10^{-5}). Initial values for the optimization were randomly chosen between 0.1 and 0.6 and led to absolute convergence for all countries except France and Italy, for which we chose initial values to be same as the final result of Mallick (2012) to obtain convergence. We found that the elasticity of substitution σ varied between 0.36 and

0.64 for all the countries examined. This is in line with the previous related research in the literature (see, for example, Mallick, 2012; Klump et al., 2007). Our estimates for the growth rate of productivity α is also very similar to results in the literature ranging from 1.5% to 2.9% for all countries except Italy where it was 4.11%. The productivity growth rate was higher for European economies compared to non-European, similar to what we observed in the case of an exponential productivity growth model of the previous two Sections. Apart from Italy, the order of magnitude is also very similar for all countries in the two methods. Average capital share μ_0 lies between 28.5% for UK to 38.5% for Norway.

Version 1 - Real Philips curve

The Phillips curve used here has an extra term for productivity growth rate, leading to a new unrestricted error correction model of the form

$$\Delta z_t = \varphi_0 + \varphi_1 \Delta \alpha_{t-1} + \varphi_2 \Delta \lambda_{t-1} + \varphi_3 z_{t-1} + \varphi_4 \lambda_{t-1} + \varphi_5 \alpha_{t-1} + \epsilon_{5t} \tag{43}$$

$$\alpha_t = \log(a_t) - \log(a_{t-1}) \tag{44}$$

and the restriction to be tested is $\varphi_3 = \varphi_4 = \varphi_5 = 0$. Table 10 gives the F-statistic of the restriction. The F-statistic is greater than the $I(1)$ threshold given in Narayan (2005) for 50 observations and $k = 2$, implying that wage growth, productivity growth and employment rate are co-integrated. Moreover, Table 11 shows that the p-values are greater than any acceptable threshold for all the countries, ensuring that the errors of the unrestricted error correction model in (43) do not suffer from auto-correlation and the model is well specified.

Having established that variables are co-integrated, we estimate the following long-run "levels model":

$$z_t = \gamma + \rho \lambda_t + \varrho \alpha_t + \epsilon_{6t} \tag{45}$$

Table 12 shows that all the countries have negative intercept and positive slopes for both employment rate and productivity growth rate. As a final check, we next present the estimates of the restricted error correction model in Table 13. The error correction terms are negative and significant for all the countries.

We next check for structural change in the data underlying the estimation of (43) and (45) using CUSUM and CUSUMSQ tests. As we can see in Figures 4 and 5, the fluctuations for the CUSUM test remain well within the 99% confidence interval for all countries, whereas the CUSUMSQ test show fluctuations falling outside the 99% confidence interval for very brief periods for Denmark, Finland and Norway only. We therefore accept the null hypothesis of constant parameters for equations (43) and (45) throughout the period.

Table 3 summarizes the results for Version 1 of the van der Ploeg model. The estimates for the parameters γ, ρ, and ϱ appearing in (23) are taken from Table 12, whereas the estimates for the parameters σ, A and μ appearing in the production function (20) are obtained from the values presented in Table 9 and the formulas (37) and (38). Using the estimates for β and δ found in the Section 1 (see Table 1), we can calculate the equilibrium values λ_P and ω_P by substituting these estimates into equations (27) and (26), that is,

$$\lambda_P = \frac{(1-\widehat{\varrho})\widehat{\alpha} - \widehat{\gamma}}{\widehat{\rho}} \tag{46}$$

$$\omega_P = 1 - \left(\frac{\widehat{\alpha} + \widehat{\beta} + \widehat{\delta}}{\widehat{A}}\right)^{1-\widehat{\sigma}} \widehat{\mu}^{\widehat{\sigma}}. \tag{47}$$

These values are shown in the last two columns of Table 3. The employment rate continue to be of the same order and within the bounds of observable data for all the countries except Canada. The more interesting observation is the improvement in estimates of wage share. For 6 out of 10 countries examined (Australia, Finland, France, Italy, Norway, UK), the estimate of wage share lie within the bounds of the observed values. Moreover, the difference between the estimated equilibrium and the observed mean has been reduced to a range of 1.5-7.5%, compared to the 2-12% percentage points range in the case of the Goodwin model estimated in Section 1. The average relative error for the equilibrium wage share across all countries was reduced from 13.6% for the Goodwin model to 6.6% for the van der Ploeg model.

[Insert Table 3 here]

Version 2 - Nominal Philips curve

The variables of interest (nominal wage growth rate, employment rate and inflation) are non-stationary for Australia, Canada, Denmark, Finland, France, Italy, UK and Germany, for which we can use Johansen's co-integration test. However, since Norway and US have stationary inflation and nominal wage growth respectively, we have to use the ARDL bounds-testing approach for these countries. Table 14 suggests that $H_0 : k \leq 0$ is rejected and $H_0 : k \leq 1$ is accepted for Australia, Canada, Denmark, Finland, France, and Germany thus implying that variables are co-integarted for these countries. However, the tests rejected the hypothesis of co-integration for Italy and UK. Using the ARDL bounds testing approach for Norway and US, we find that their F-statistics are 5.21 and 5.65, respectively. Both of these values are higher than the critical values given in Narayan (2005) at the 10% significance level.

Next we present the estimates for γ, ρ and η for all the countries in Table 4. For Australia, Canada, Denmark, Finland, France, and Germany we get the estimate using Johansen's maximum likelihood estimates (MLE). For the rest, we present the ordinary least-squares (OLS) estimates. OLS estimates are consistent for Norway and US since we have shown that the variables are co-integrated. For sake of completeness, we also present the OLS estimates for Italy and UK. The estimates of the parameter η (impact of inflation on bargaining) is less than or equal to one for all countries except Germany and this is very similar to the slope of expected inflation observed in Desai's model of the previous section (see estimate of η in Table 2).

Using the estimates for β and δ found in the Section 1 (see Table 1, the estimates for σ, μ and A found in the previous section (see Table 3) and the estimates χ and π found in Section 2.3 (see Table 2), we can calculate the equilibrium values for Version 2 of the van der Ploeg model by substituting these parameter values in (32) and (33), that is,

$$\omega_{PE} = 1 - \left(\frac{\widehat{\alpha} + \widehat{\beta} + \widehat{\delta}}{\widehat{A}} \right)^{1-\widehat{\sigma}} \widehat{\mu}^{\widehat{\sigma}} \tag{48}$$

$$\lambda_{PE} = \frac{\widehat{\alpha} - \widehat{\gamma} + (1 - \widehat{\eta})\widehat{\chi} \log(\omega_{PE}\widehat{\pi})}{\widehat{\rho}}. \tag{49}$$

The results are shown in the last two columns of Table 4. As is evident from the graphs in Figure 1, the estimate of employment rate for both versions of the model have a very similar range. The difference in the two estimates of employment rate ranges between -2.27% for Canada to 2.33% for US. The equilibrium wage share estimates for the two versions of the model are, of course, the same.

[Insert Table 4 here]

4 Concluding remarks

As can be seen in Table 5, the errors in equilibrium estimates for employment rate in both the Desai and van der Ploeg models are comparable to those in the original Goodwin model, with the original model performing slightly better than both extensions. As mentioned above, it is in the estimation for equilibrium wage share that the less restrictive assumptions of the van der Ploeg model lead to larger improvement: Table 6 shows the average relative error between the empirical and estimated equilibrium wage share reduced to 6.59%, down from 13.58% in the Goodwin model.

[Insert Table 5 here]

[Insert Table 6 here]

[Insert Figure 1 here]

References

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A Auxiliary Tables

[Insert Tables 7 to 14 here]

B Auxiliary Figures

[Insert Figures 4 to 5 here]

Country	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\delta}$	$\widehat{\nu}$	$\widehat{\gamma}$	$\widehat{\rho}$	ω_G	λ_G	T_G
Australia	0.0147	0.0197	0.0522	2.881	-0.215	0.242	0.7506	0.9480	25.70
Canada	0.0126	0.0201	0.0429	2.864	-0.095	0.115	0.7836	0.9371	36.58
Denmark	0.0181	0.0058	0.0497	2.842	-0.330	0.367	0.7907	0.9492	20.18
Finland	0.0288	0.0030	0.0519	3.314	-0.258	0.303	0.7224	0.9480	25.13
France	0.0221	0.0076	0.0377	3.326	-0.491	0.549	0.7756	0.9346	18.17
Germany	0.0270	0.0060	0.0362	3.358	-0.699	0.747	0.7678	0.9717	15.42
Italy	0.0208	0.0056	0.0466	3.206	-0.891	0.982	0.7663	0.9285	13.46
Norway	0.0228	0.0114	0.0465	3.208	-0.574	0.609	0.7411	0.9804	16.92
UK	0.0205	0.0052	0.0372	3.053	-0.108	0.135	0.8078	0.9515	34.10
US	0.0155	0.0165	0.0521	2.725	-0.227	0.257	0.7708	0.9441	24.00

Table 1: Parameter estimates and implied equilibrium values for the Goodwin model.

Country	$\widehat{\gamma}$	$\widehat{\rho}$	$\widehat{\eta}$	$\widehat{\zeta}$	$\widehat{\chi}$	$\widehat{\pi}$	$\widehat{\kappa}$	ω_D	λ_D
Australia	-0.3160	0.3498	0.9975	0.6913	0.3797	1.673	0.4173	0.7342	0.9462
Canada	-0.2291	0.2611	0.9593	0.6338	0.3918	1.589	1.6353	0.7474	0.9361
Denmark	-0.3106	0.3477	0.9530	0.8740	0.4971	1.536	0.8374	0.7708	0.9567
Finland	-0.3516	0.4083	0.9005	0.6782	0.3028	1.620	1.2289	0.7304	0.9443
France	-0.4956	0.5562	0.9469	1.0216	0.3787	1.522	1.4090	0.7781	0.9370
Germany	-0.6824	0.7288	1.0241	0.9952	0.3481	1.569	1.8129	0.7812	0.9710
Italy	-0.9196	1.0130	0.9925	0.8458	0.4355	1.609	0.5095	0.7729	0.9290
Norway	-1.2375	1.3284	0.2976	0.4796	0.1414	2.240	0.4612	0.7565	0.9881
UK	-0.1124	0.1424	0.9539	0.9923	0.6319	1.447	1.1622	0.8030	0.9641
US	-0.2452	0.2831	0.8181	0.7130	0.4501	1.595	1.4115	0.7356	0.9673

Table 2: Parameter estimates and implied equilibrium values for the Desai model.

Country	$\widehat{\gamma}$	$\widehat{\rho}$	$\widehat{\varrho}$	$\widehat{\sigma}$	$\widehat{\mu}$	$\widehat{A}$	ω_P	λ_P
Australia	-0.221	0.245	0.232	0.377	0.033	0.083	0.716	0.950
Canada	-0.078	0.093	0.192	0.375	0.023	0.072	0.742	0.964
Denmark	-0.297	0.323	0.447	0.587	0.300	0.339	0.733	0.950
Finland	-0.246	0.285	0.169	0.424	0.020	0.043	0.714	0.948
France	-0.384	0.428	0.256	0.361	0.011	0.047	0.745	0.942
Germany	-0.445	0.470	0.575	0.521	0.061	0.050	0.726	0.973
Italy	-0.467	0.509	0.625	0.602	0.101	0.057	0.690	0.947
Norway	-0.571	0.602	0.198	0.391	0.534	0.388	0.689	0.986
UK	-0.118	0.138	0.376	0.596	0.064	0.037	0.754	0.958
US	-0.246	0.265	0.626	0.647	0.162	0.094	0.698	0.951

Table 3: Parameter estimates and implied equilibrium values for Version 1 of the van der Ploeg Model

Country	$\widehat{\gamma}$	$\widehat{\rho}$	$\widehat{\eta}$	ω_{PE}	λ_{PE}
Australia	-0.143	0.178	0.790	0.716	0.969
Canada	-0.255	0.288	0.985	0.742	0.942
Denmark	-0.354	0.390	1.007	0.733	0.954
Finland	-0.112	0.142	1.068	0.714	0.965
France	-0.511	0.573	0.911	0.745	0.943
Germany	-0.681	0.716	1.334	0.726	0.968
Italy	-0.901	0.994	0.981	0.690	0.949
Norway	-1.151	1.245	0.184	0.689	0.986
UK	-0.116	0.146	0.951	0.754	0.964
US	-0.159	0.193	0.776	0.698	0.975

Table 4: Parameter estimates and implied equilibrium values for Version 2 of the van der Ploeg model

	Goodwin		Desai		van der Ploeg 1		van der Ploeg 2	
	$\lvert\bar{\lambda}-\lambda_G\rvert$	$\frac{\lvert\bar{\lambda}-\lambda_G\rvert}{\bar{\lambda}}$	$\lvert\bar{\lambda}-\lambda_D\rvert$	$\frac{\lvert\bar{\lambda}-\lambda_D\rvert}{\bar{\lambda}}$	$\lvert\bar{\lambda}-\lambda_P\rvert$	$\frac{\lvert\bar{\lambda}-\lambda_P\rvert}{\bar{\lambda}}$	$\lvert\bar{\lambda}-\lambda_{PE}\rvert$	$\frac{\lvert\bar{\lambda}-\lambda_{PE}\rvert}{\bar{\lambda}}$
Australia	0.0023	0.24%	0.0005	0.05%	0.0043	0.45%	0.0233	2.46%
Canada	0.0107	1.16%	0.0097	1.05%	0.0376	4.06%	0.0156	1.68%
Denmark	0.0062	0.65%	0.0013	0.13%	0.0054	0.57%	0.0014	0.15%
Finland	0.0105	1.12%	0.0068	0.73%	0.0105	1.12%	0.0275	2.93%
France	0.0015	0.16%	0.0009	0.10%	0.0059	0.63%	0.0069	0.74%
Germany	0.0002	0.02%	0.0009	0.09%	0.0011	0.11%	0.0039	0.40%
Italy	0.0005	0.05%	0.0010	0.11%	0.0190	2.05%	0.0210	2.26%
Norway	0.0073	0.75%	0.0150	1.54%	0.0129	1.33%	0.0129	1.33%
United Kingdom	0.0077	0.82%	0.0203	2.15%	0.0142	1.50%	0.0202	2.14%
United States	0.0025	0.27%	0.0257	2.73%	0.0094	1.00%	0.0334	3.55%
Average	0.0049	0.52%	0.0082	0.87%	0.0120	1.28%	0.0166	1.76%

Table 5: Comparison between errors in equilibrium values estimates for employment rate in the Goodwin, Desai, and van der Ploeg models - 1960 to 2010.

	Goodwin		Desai		van der Ploeg	
	$\lvert\overline{\omega}-\omega_G\rvert$	$\frac{\lvert\overline{\omega}-\omega_G\rvert}{\overline{\omega}}$	$\lvert\overline{\omega}-\omega_D\rvert$	$\frac{\lvert\overline{\omega}-\omega_D\rvert}{\overline{\omega}}$	$\lvert\overline{\omega}-\omega_P\rvert$	$\frac{\lvert\overline{\omega}-\omega_P\rvert}{\overline{\omega}}$
Australia	0.099	15.18%	0.082	12.66%	0.064	9.87%
Canada	0.111	16.54%	0.075	11.16%	0.070	10.35%
Denmark	0.106	15.55%	0.086	12.63%	0.049	7.12%
Finland	0.023	3.24%	0.034	4.86%	0.014	2.04%
France	0.066	9.33%	0.069	9.69%	0.037	5.02%
Germany	0.084	12.28%	0.097	14.24%	0.042	6.17%
Italy	0.085	12.46%	0.092	13.43%	0.009	1.26%
Norway	0.126	20.54%	0.142	23.04%	0.074	12.07%
United Kingdom	0.093	13.03%	0.088	12.36%	0.039	5.50%
United States	0.116	17.64%	0.080	12.27%	0.043	6.53%
Average	0.091	13.58%	0.085	12.63%	0.044	6.59%

Table 6: Comparison between errors in equilibrium values estimates for wage share in the Goodwin, Desai, and van der Ploeg models - 1960 to 2010.

Country	Variable	$\widehat{\gamma}$	$\widehat{\rho}$	$\widehat{\eta}$	$\widehat{\zeta}$	AdjR2	$\zeta = 1$	$\eta = 1$	$\zeta = 1$ & $\eta = 1$
Australia	Coeff	-0.316	0.350	0.997	0.691	0.523			
	pValue	0.033	0.026	0.000	0.000		0.008	0.982	0.027
Canada	Coeff	-0.229	0.261	0.959	0.634	0.624			
	pValue	0.061	0.048	0.000	0.000		0.000	0.664	0.000
Denmark	Coeff	-0.311	0.348	0.953	0.874	0.475			
	pValue	0.005	0.003	0.000	0.000		0.335	0.600	0.542
Finland	Coeff	-0.352	0.408	0.901	0.678	0.556			
	pValue	0.001	0.000	0.000	0.000		0.003	0.387	0.004
France	Coeff	-0.496	0.556	0.947	1.022	0.611			
	pValue	0.000	0.000	0.000	0.000		0.856	0.263	0.522
Germany	Coeff	-0.682	0.729	1.024	0.995	0.669			
	pValue	0.000	0.000	0.000	0.000		0.975	0.885	0.989
Italy	Coeff	-0.920	1.013	0.992	0.846	0.583			
	pValue	0.000	0.000	0.000	0.000		0.188	0.904	0.404
Norway	Coeff	-1.237	1.328	0.298	0.480	0.350			
	pValue	0.000	0.000	0.060	0.000		0.000	0.000	0.000
UK	Coeff	-0.112	0.142	0.954	0.992	0.764			
	pValue	0.150	0.088	0.000	0.000		0.928	0.378	0.649
US	Coeff	-0.245	0.283	0.818	0.713	0.466			
	pValue	0.045	0.029	0.000	0.000		0.017	0.076	0.005

Table 7: Estimation of expectation-augmented Phillips Curve given by equation (17). The columns labeled "$\zeta = 1$" and "$\eta = 1$" contain p-values for the t-statistic for the test, whereas column labeled "$\zeta = 1$ & $\eta = 1$" gives the p-values for the F-test.

Country	Variable	$\widehat{\chi \log \pi}$	$\widehat{\chi}$	AdjR2	$\widehat{\pi}$
Australia	Coeff	0.195	0.380	0.744	1.673
	pValue	0.000	0.000		
Canada	Coeff	0.181	0.392	0.393	1.589
	pValue	0.000	0.000		
Denmark	Coeff	0.213	0.497	0.748	1.536
	pValue	0.000	0.000		
Finland	Coeff	0.146	0.303	0.600	1.620
	pValue	0.000	0.000		
France	Coeff	0.159	0.379	0.851	1.522
	pValue	0.000	0.000		
Germany	Coeff	0.157	0.348	0.648	1.569
	pValue	0.000	0.000		
Italy	Coeff	0.207	0.435	0.749	1.609
	pValue	0.000	0.000		
Norway	Coeff	0.114	0.141	0.135	2.240
	pValue	0.000	0.005		
UK	Coeff	0.233	0.632	0.888	1.447
	pValue	0.000	0.000		
US	Coeff	0.210	0.450	0.671	1.595
	pValue	0.000	0.000		

Table 8: Markup equation (18)

Country	$\widehat{\sigma}$	$\widehat{\alpha}$	$\widehat{\mu_0}$	$\widehat{\frac{K_0}{\xi Y_0}}$
Australia	0.377	1.53%	34.8%	2.86
Canada	0.375	1.53%	32.8%	2.86
Denmark	0.587	1.90%	31.6%	2.81
Finland	0.424	2.88%	30.0%	3.30
France	0.361	2.60%	29.1%	3.36
Germany	0.521	2.79%	31.6%	3.42
Italy	0.602	4.11%	31.9%	3.27
Norway	0.391	2.68%	38.5%	3.20
UK	0.596	2.26%	28.5%	3.04
US	0.647	1.83%	34.5%	2.71

Table 9: Estimates for the production function (35)

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
F statistics	10.00	13.52	20.60	15.61	9.35	11.15	15.37	6.88	5.51	5.94

Table 10: F-test for $H_0 : \varphi_3 = \varphi_4 = \varphi_5 = 0$ in equation (43). Lower and upper bounds for $I(0)$ and $I(1)$ at the 1%, 5% and 10% levels are [7.560, 8.685], [5.220, 6.070] and [4.190, 4.940], respectively.

Country	lag 1	lag 2	lag 3	lag 4	lag 5
Australia	0.971	0.958	0.909	0.918	0.935
Canada	0.732	0.204	0.305	0.458	0.596
Denmark	0.674	0.901	0.727	0.806	0.864
Finland	0.746	0.534	0.435	0.602	0.688
France	0.700	0.926	0.779	0.635	0.585
Germany	0.902	0.828	0.922	0.728	0.580
Italy	0.232	0.214	0.356	0.517	0.643
Norway	0.820	0.643	0.648	0.799	0.886
UK	0.919	0.994	0.941	0.411	0.508
US	0.714	0.872	0.943	0.924	0.538

Table 11: p-values for the alternative hypothesis that the errors are AR(m) for $m = 1, \ldots, 5$ in the serial correlation test for Unrestricted Error Correction Model given by equation (43)

Country	Variable	$\widehat{\gamma}$	$\widehat{\rho}$	$\widehat{\varrho}$	AdjR2
Australia	Coeff	-0.221	0.245	0.232	0.105
	pValue	0.025	0.019	0.166	
Canada	Coeff	-0.078	0.093	0.192	0.023
	pValue	0.382	0.333	0.205	
Denmark	Coeff	-0.297	0.323	0.447	0.386
	pValue	0.001	0.000	0.000	
Finland	Coeff	-0.246	0.285	0.169	0.287
	pValue	0.000	0.000	0.176	
France	Coeff	-0.384	0.428	0.256	0.772
	pValue	0.000	0.000	0.039	
Germany	Coeff	-0.445	0.470	0.575	0.804
	pValue	0.000	0.000	0.000	
Italy	Coeff	-0.467	0.509	0.625	0.788
	pValue	0.000	0.000	0.000	
Norway	Coeff	-0.571	0.602	0.198	0.029
	pValue	0.104	0.096	0.479	
UK	Coeff	-0.118	0.138	0.376	0.214
	pValue	0.070	0.046	0.002	
US	Coeff	-0.246	0.265	0.626	0.607
	pValue	0.001	0.001	0.000	

Table 12: Long term estimates of Phillips Curve parameters in equation (45)

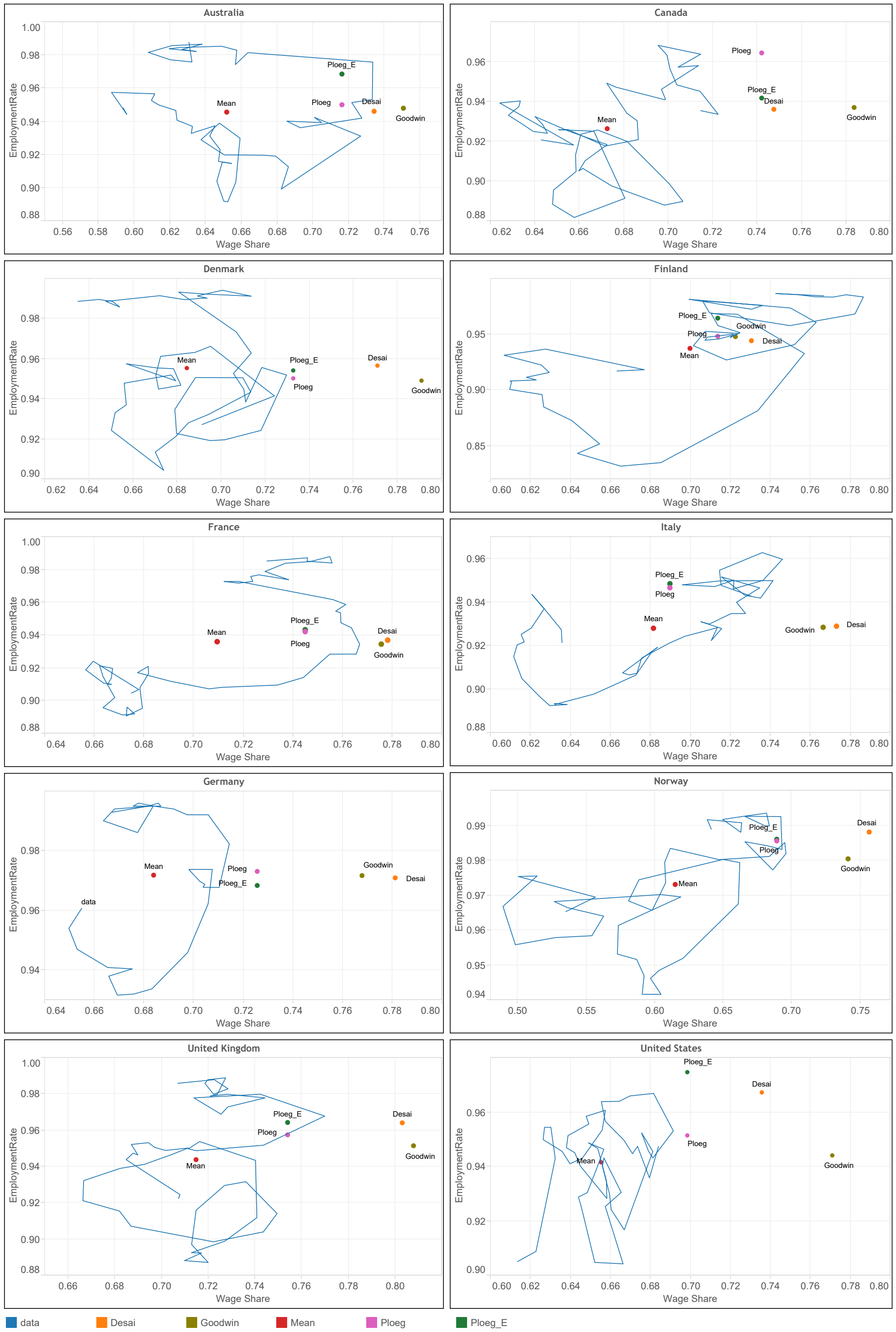
Country	Variable	constant	$\Delta\alpha_t$	$\Delta\lambda_t$	$\hat{\epsilon}_{6(t-1)}$	AdjR2
Australia	Coeff	0.000	0.072	0.187	-0.846	0.412
	pValue	0.984	0.579	0.597	0.000	
Canada	Coeff	0.000	0.122	-0.044	-0.921	0.418
	pValue	0.827	0.393	0.852	0.000	
Denmark	Coeff	-0.001	-0.146	0.191	-1.245	0.524
	pValue	0.677	0.160	0.481	0.000	
Finland	Coeff	0.000	-0.011	0.346	-1.002	0.494
	pValue	0.945	0.932	0.096	0.000	
France	Coeff	-0.002	-0.064	-0.036	-0.793	0.348
	pValue	0.249	0.538	0.882	0.000	
Germany	Coeff	-0.002	-0.174	0.118	-1.246	0.342
	pValue	0.545	0.242	0.782	0.000	
Italy	Coeff	-0.002	-0.336	-0.184	-1.147	0.430
	pValue	0.433	0.004	0.667	0.000	
Norway	Coeff	-0.002	-0.484	0.410	-0.994	0.498
	pValue	0.762	0.108	0.711	0.000	
UK	Coeff	0.000	-0.238	0.032	-0.720	0.330
	pValue	0.890	0.044	0.906	0.000	
US	Coeff	-0.001	-0.197	-0.446	-0.822	0.280
	pValue	0.670	0.054	0.015	0.001	

Table 13: Restricted Error-correction Model

Country	Australia	Canada	Denmark	Finland	France	Italy	UK	Germany	cValue
k=0	40.07	44.23	53.60	44.71	39.07	22.77*	26.09*	31.18	29.80
k=1	13.10*	12.53*	13.85*	14.87*	13.98*	12.67	9.57	11.99*	15.49
k=2	5.25	4.32	2.13	4.38	5.52	4.47	3.82	2.86	3.84

Table 14: Johansen's test for co-integration (* indicates the value of r for which null hypothesis was accepted)

Figure 1: Estimated equilibrium and observed mean for employment rate and wage share



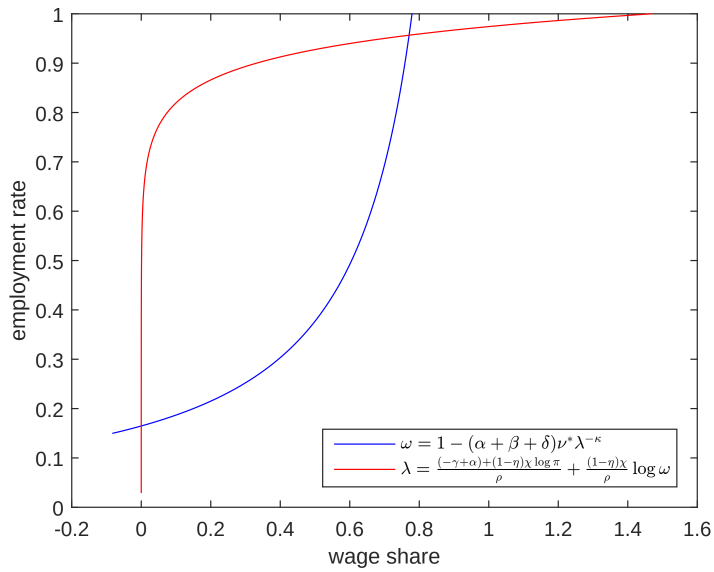
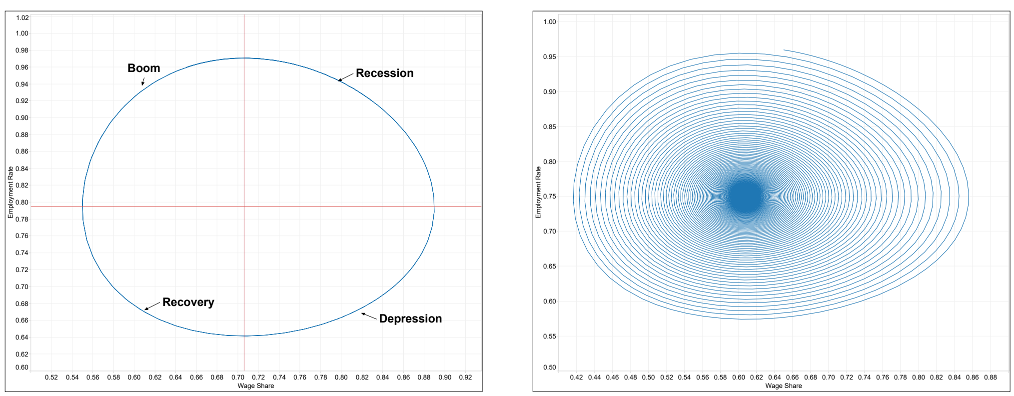
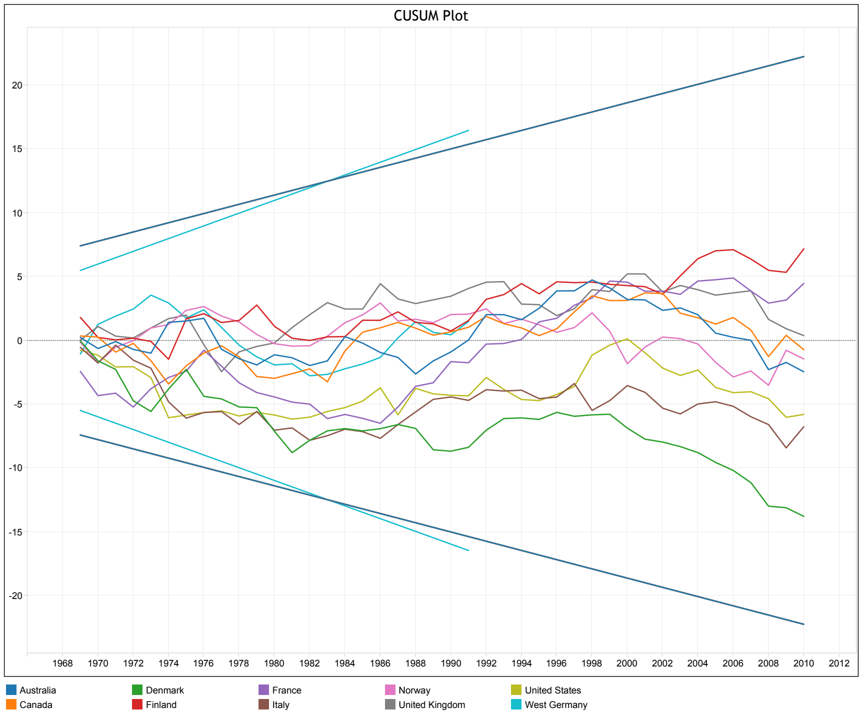


Figure 2: Equilibrium values for the Desai model. Parameter values used: $\gamma = -0.3106$, $\rho = 0.3477$, $\eta = 0.9530$, $\alpha = 0.0181$, $\beta = 0.0058$, $\kappa = 0.8374$, $\pi = 1.5357$, $\delta = 0.0497$, $\chi = 0.497$, $\zeta = 0.874$, $\nu* = 3$



(a) Goodwin model (b) van der Ploeg model

Figure 3: Solutions for the Goodwin model with parameter values $\alpha = 0.018$, $\beta = 0.02$, $\delta = 0.06$, $\gamma = 0.3$, $\rho = 0.4$, $\nu = 3$ and the van der Ploeg model (26)-(27) with the same base parameters and $\sigma = 0.005$, $\mu = 0.5$ and $A = 0.25$



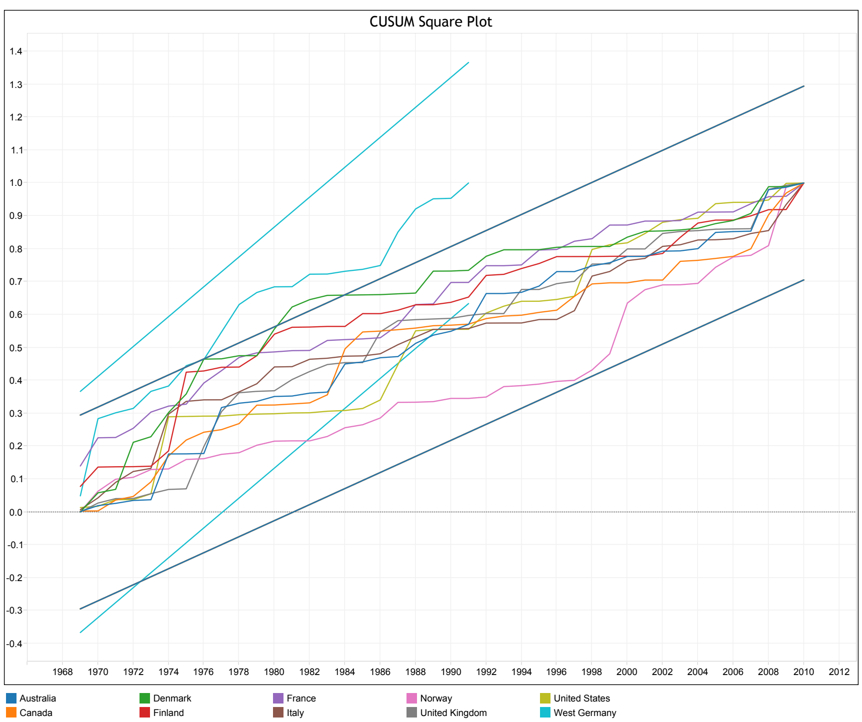
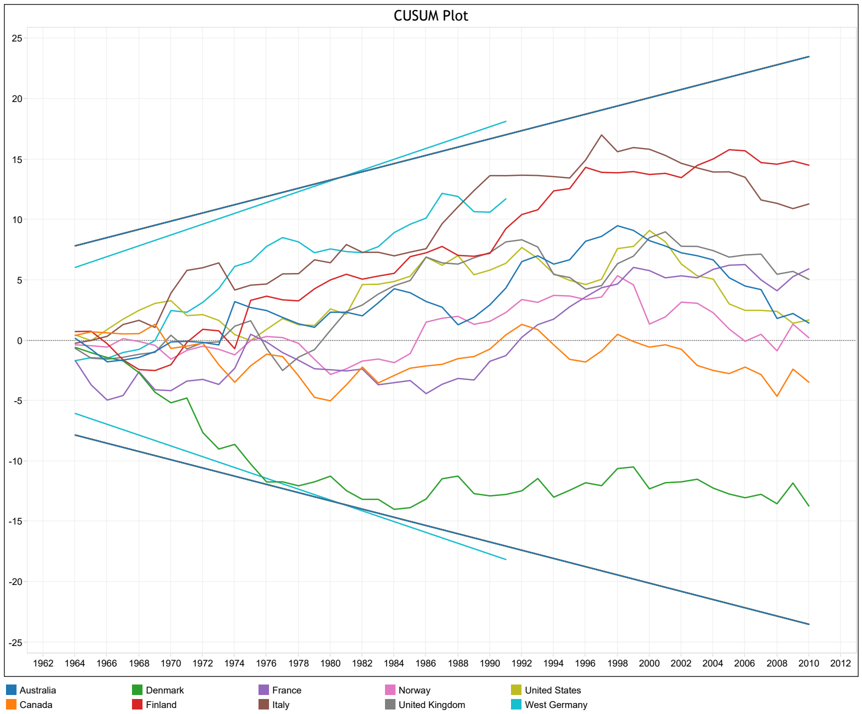


Figure 4: Tests for structural changes in (43) using CUSUM and CUSUMSQ tests at the 99% confidence interval.



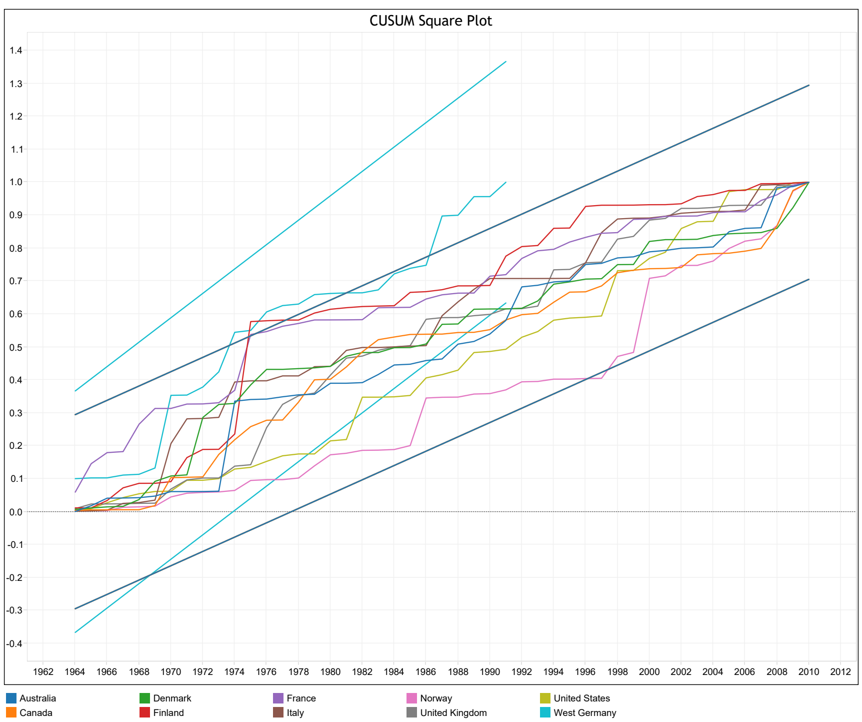


Figure 5: Tests for structural changes in (45) using CUSUM and CUSUMSQ tests at the 99% confidence interval.