Lecture #4

Optimal Control of McKean-Vlasov Equations (Mean Field Control) and the Price of Anarchy

We introduce the optimal control of stochastic dynamics of the McKean-Vlasov type (also known as Mean Field Control) and we identify the solution as the result of the optimization of the social cost for a central planner controlling a population of individuals interacting in a mean field manner. We compare the result of this optimization to the costs incurred by players in a Mean Field Game Nash equilibrium. During the second part of the lecture, we quantify the efficiency (or lack thereof) of Mean Field Games (MFGs) through a natural extension of the concept of Price of Anarchy (PoA). We also propose ways to incentivize players, in the spirit of the theory of mechanism design, for them to adopt an optimal behavior or generate an optimal value. In the process, we define a notion of price of instability (PoI) for the social welfare equilibrium as identified by the solution of a central planner optimization problem.