Math 4A03 Real Analysis II (taught by Patrick Speissegger in term 1 of 2017–18)

Analysis starts with the fundamental concept of distance: the properties of space are largely determined by how distances are measured, and different measures of distance give rise to very different worlds. From the notion of distance we may then extend the calculus tools of limits, continuity, and smoothness to new and different “spaces”. Much of what we will cover involves infinite dimensional spaces, whose “points” are not numbers or vectors, but (for example) functions defined on a common domain. With these methods we may study approximation of functions (by polynomials or Fourier Series) and the solution of equations (integral, differential, or nonlinear.) Real analysis provides the foundation (and often the underlying toolbox) for many mathematical areas where limits and continuity play an important role, such as complex variables (3X03, 4X03), differential equations (3F03, 3FF3), dynamical systems (3DC3), Fourier series and transforms, numerical analysis (3NA3 and 4NA3), probability and stochastic processes (Stats 3U03), and topology (3T03, 4B03). The material of this course will be of interest to anyone desiring a deeper understanding of the underlying analysis related to the above areas. It is strongly recommended for any student who is considering graduate school.

Math 4AT3 Topics in Analysis (not taught in 2017–18)

Math 4B03 Calculus on Manifolds (taught by Hans Boden in term 1 in 2017–18)

The highlight of vector calculus consists of the theorems of Stokes and Gauss together with their numerous applications. Especially noteworthy from a theoretical perspective are the hints these theorems provide regarding the existence of “holes” in regions in space and their detection via the presence of of curl-free (respectively divergence-free) vector fields which cannot be derived from any scalar (respectively vector) potentials. On the application front, it is most striking that the basic laws of electromagnetism (Maxwell’s equations) in integral form are expressed using these two theorems. All this is an illustration of the inextricable relationship between Geometry and Physics.

In Math 4B3/6B3 the mathematical structures underlying the theorems of Stokes and Gauss will be revealed and generalised, leading to Stokes’ theorem for manifolds, which is the logo on the home page of the Department of Mathematics and Statistics. It will be shown that differential forms, which generalise the gradient, curl, and divergence, are exactly the right objects to work with on a manifold in order to have a geometric (and intrinsic) description of classical mechanics and electromagnetism. Furthermore, differential forms which satisfy suitable linear partial differential equations can also be used to study properties of the underlying smooth manifolds which are invariant under continuous or smooth deformations.

All one needs to participate in this exciting journey of discovery is a strong grasp of advanced calculus and linear algebra. In particular, there is no need to be familiar with physics, the geometry of surfaces, or general topology.

Math 4BT3 Topics in Geometry (not taught in 2017–18)

Math 4E03 Galois Theory (not taught in 2017–18)

Math 4ET3 Topics in Algebra (not taught in 2017–18)

Math 4FM3 Financial Markets and Derivatives (taught by Traian Pirvu in term 2 of 2017–18)
Math 4FM3 blends together elements of stochastic calculus, practical issues in economics and the financial markets, and professional skills, into a course designed for launching students into professional careers upon graduation, or into a graduate program in financial mathematics. Following from Math 3FM3, the course develops continuous time models for stocks, foreign exchange rates, and interest rates in order to tackle the mathematical challenges of pricing and hedging derivative instruments in these markets. The models employ Brownian motion/geometric Brownian motion, leading to the introduction of Ito calculus to construct portfolios that protect the user from risks in the financial markets. The course develops, explores, and extends the famous Black-Scholes model, and many famous models for fixed income derivatives.

Math 4FM3 develops the mathematics in the context of real-world issues, and aims to develop students' professional competencies. Each class begins with students presenting articles they have read through news sources, and the class analyzes its impact and relevance to the material being studied. Students complete the course with group projects on topics in financial mathematics of current interest in the financial industry, delivering a short presentation and paper. This complements assessment using a midterm, final exam, class participation marks, and assignments. The assignments include introductions and tutorials in the use of Excel and Visual Basic.

Math 4FM3 serves as the capstone course to the University’s Actuarial and Financial Mathematics program, and together with Math 3FM3, completes the study of material examined in the actuarial MFE exam. But the course primarily aims to develop students’ skills in the practice of taking advanced mathematical methods, and applying them to current problems being tackled in the financial industry and by academic researchers.

**Math 4FT3 Topics in Differential Equations** (taught by Walter Craig in 2017–18)

Does entropy really increase no matter what we do? Can light pass through a Big Bang? What is certain about the Heisenberg uncertainty principle?

Many of the laws of physics are formulated in terms of differential equations, and the questions above are about the nature of their solutions. The course Math 4FT3 addresses the principal partial differential equations that arise in physical contexts; the course material will present methods (both specific and abstract) for solving them. In the course of this analysis we will deduce many of the main features of their solutions, including finite propagation speed and conservation of energy for wave equations, maximum principles for heat flow, shock formation for nonlinear conservation laws, and other mathematical aspects of familiar and less familiar laws of physics. No knowledge of physics is assumed.

**Math 4GR3 Groups and Rings** (taught by Matthew Valeriote in term 2 of 2017–18)

Further topics in group theory and ring theory. Topics include: direct products, Fundamental Theorem of Finite Abelian Groups, Sylow Theorems, free groups, group presentations, fields and integral domains, special integral domains (Euclidean, principal ideal, unique factorization), fields of fractions of integral domains, polynomial rings in many variables, and additional topics at the discretion of the instructor (e.g., Groebner bases, algebraic coding theory).

**Math 4L03 Introduction to Mathematical Logic** (not taught in 2017–18)

**Math 4LT3 Topics in Logic: theory of computation** (not taught in 2017–18)

**Math 4MB3 Mathematical Biology** (taught by David Earn in term 2 of 2017–18)

The goal of Math 4MB3/6MB3 is to use various mathematical and statistical modelling approaches to better understand biological systems. The course focuses mainly on modelling disease
spread through time, but also discusses topics in ecology and evolutionary biology. We first dis-
cuss various modelling approaches for short time periods in the context of a single epidemic. We
then approach the more complex problem of modelling recurring epidemics. Lastly, we approach
spatio-temporal modelling. The modelling techniques presented are discussed in the context of the
mathematics underlying them and of their meaning in a biological context.

Another goal of the course is to introduce students to primary literature in mathematical biology.
The prerequisites for the course are a basic knowledge of the qualitative theory of differential
equations (Math 3F03) and an interest in applying those techniques to models of biological systems.
The course is considered to be the capstone course for the math stream of the combined honours
program in Biology and Mathematics, but no particular material about biology will be assumed.

**Math 4NA3 Numerical Methods for Differential Equations** (taught by Matheus Grasselli
in term 2 in 2017–18)

**Math 4P06 Senior Research Project** (students find an advisor)

Before the course starts:

1. A student identifies a supervisor and a theme/topic of their research.

2. The supervisor and the student discuss the course and course expectations in detail, including:
   - Research theme/topic (of course, can be/will be modified as time progresses), background
     reading, references, software, etc.
   - Logistics of term work: how often will the meetings between a supervisor and a students
     be held, what is expected of the student at these meetings
   - Benchmarks and corresponding deadlines (for instance, deadlines for background reading,
     data collection, data analysis, deadlines for drafts and/or parts of the thesis to be handed
     in)
   - Details about how the final course mark will be assigned (it is desirable that part of the
     grade, say up to 20%, is given for the oral presentation). This part should be as specific
     as possible, for both student’s and supervisor’s protection.

3. Written thesis and final oral presentation are integral parts of the course.

4. After an agreement on items (2), and possibly other items has been reached, a supervisor and
   a student fill out and sign the 4P06 form (downloadable from the Departments web page).

5. Once the form is approved by the Department, the student formally registers for the course.

   Terms 1 and 2:

   6. Supervisor and student meet on a regular basis (say, once every 2-3 weeks in low-intensity
      periods, and as often as needed otherwise); this way, supervisor can monitor students progress
      and give valuable feedback to the student. As well, it helps the student manage their time
      more efficiently.

**Math 4Q03 Numerical Methods for Differential Equations** (see Math 4NA3)

**Math 4TT3 Topics in Topology** (not taught in 2017–18)
Math 4W06, 4WW6 Reading in Mathematics I and II (students find an advisor)

Are you interested in some area or problem in Math or Stats but those have not been covered in the courses you took? Approach a faculty (or a postdoc) with your idea/ interest/ suggestion and discuss this one-on-one opportunity. The area of study in these courses has to be different from the material covered in the courses offered in Math and Stats.

You need to:

- Identify an area that you want to work on; as well, identify a faculty supervisor with whom you will be working.

- Discuss arrangements with your chosen supervisor, which need to include: objectives (what are you supposed to accomplish? study a text, or a paper, read a book, etc.); deliverables (what do you need to do? weekly oral reports, written reports, assignments, final project, etc.); details on how the course mark will be assigned at the end.

All the above should be presented in the standard format of a course outline. You and your supervisor fill out the form and submit it to the Associate Chair for approval. Once signed, the form is processed by the Math and Stats office, and you go ahead and formally register for the course.

Note: These courses can be taken in any term (provided that you identify a supervisor willing to work with you)

Math 4X03 Complex Analysis II (not taught in 2017–18)

Stats 4A03 Time Series (taught by Roman Viveros in term 2 of 2017–18)

Time series is comprised of a collection of models and methods to analyze data taken consecutively (hopefully) at equally spaced time points for which there is correlation. This correlation is called serial correlation or more often, autocorrelation. There are many areas of application, including economics, finance, environment, ecology and engineering. For instance, share prices in successive days, company profits in successive quarters, monthly temperature averages in a city, and the yearly abundance of rabbits in a region. Upon successful completion of this course, the students will be able to perform the following tasks: (a) Produce, interpret and explain time plots, including identification of trends and seasonality, for a time series dataset; (b) Calculate, plot and interpret autocorrelation functions; (c) Fit and interpret time series models such as ARMA (autoregressive moving average) and ARIMA (autoregressive integrated moving average) to time series data; (d) Carry out forecasts from time series data and interpret and explain the results; (e) State and justify mathematically the main theoretical tools, apply the methods numerically to datasets, and interpret and explain the results; and (e) Handle the numerical calculations and plotting in the computer using statistical software such as R.

The course prerequisites are STATS 3A03 and STATS 3D03.

Stats 4C03 Generalized Linear Models (taught by Roman Viveros in term 1 of 2017–18)

While the normal linear regression model has been studied in many courses (most prominently in STATS 3A03), there are many statistical datasets where the distribution of the response variable is non-normal. For instance, the distribution of counts is more likely to be binomial or Poisson, while the distribution of failure times for products or survival times of patients with a certain condition is more likely to be gamma or lognormal. The regression models appropriate to these situations are called generalized linear models. The model structures, including link functions, will be
presented along with the properties and estimation methods. Extensive use of R will be made to fit the models to a variety of datasets.

The course prerequisites are STATS 3A03 and STATS 3D03.

Stats 4CI3 Computational Methods for Inference (not taught in 2017–18)

Stats 4D03 Intermediate Probability Theory (not taught in 2017–18)

Stats 4I03 Inference (not taught in 2017–18)

Stats 4M03 Multivariate Analysis (taught by Sharon McNicholas in term 1 of 2017–18)

With the vast majority of modern data sets having many variables, a course in multivariate analysis is a must for anyone wishing to work or engage in further study in statistics. A multitude of interesting approaches are available to tackle multivariate data and almost all rely, to a lesser or greater extent, on computation. These include approaches for dimension reduction (e.g., principal components analysis and factor analysis), approaches for classification (e.g., discriminant analysis and logistic regression), and clustering approaches (e.g., hierarchical clustering and model-based clustering). Implementation will be discussed in addition to theory, e.g., the expectation-maximization algorithm for the factor analysis model. Several real data sets will be considered throughout the course, including high-dimensional data and what might be considered big data.

Stats 4P03 Advanced Applied Statistics (not taught in 2017–18)

Stats 4T06 Senior Research Project (students find an advisor; the course will appear in 2018/2019 Calendar; until then, students sign up for Math 4P06)

Before the course starts:

1. A student identifies a supervisor and a theme/topic of their research.

2. The supervisor and the student discuss the course and course expectations in detail, including:
   - Research theme/topic (of course, can be/will be modified as time progresses), background reading, references, software, etc.
   - Logistics of term work: how often will the meetings between a supervisor and a student be held, what is expected of the student at these meetings
   - Benchmarks and corresponding deadlines (for instance, deadlines for background reading, data collection, data analysis, deadlines for drafts and/or parts of the thesis to be handed in)
   - Details about how the final course mark will be assigned (it is desirable that part of the grade, say up to 20%, is given for the oral presentation). This part should be as specific as possible, for both student’s and supervisor’s protection.

3. Written thesis and final oral presentation are integral parts of the course.

4. After an agreement on items (2), and possibly other items has been reached, a supervisor and a student fill out and sign the 4P06 form (downloadable from the Departments web page).

5. Once the form is approved by the Department, the student formally registers for the course.

Terms 1 and 2:
6. Supervisor and student meet on a regular basis (say, once every 2-3 weeks in low-intensity periods, and as often as needed otherwise); this way, supervisor can monitor students progress and give valuable feedback to the student. As well, it helps the student manage their time more efficiently.

**Stats 4W06 Reading in Statistics** (students find an advisor)

Are you interested in some area or problem in Math or Stats but those have not been covered in the courses you took? Approach a faculty (or a postdoc) with your idea/ interest/ suggestion and discuss this one-on-one opportunity. The area of study in these courses has to be different from the material covered in the courses offered in Math and Stats.

You need to:

- Identify an area that you want to work on; as well, identify a faculty supervisor with whom you will be working.

- Discuss arrangements with your chosen supervisor, which need to include: objectives (what are you supposed to accomplish? study a text, or a paper, read a book, etc.); deliverables (what do you need to do? weekly oral reports, written reports, assignments, final project, etc.); details on how the course mark will be assigned at the end.

All the above should be presented in the standard format of a course outline. You and your supervisor fill out the form and submit it to the Associate Chair for approval. Once signed, the form is processed by the Math and Stats office, and you go ahead and formally register for the course.

Note: This course can be taken in any term (provided that you identify a supervisor willing to work with you).