Math 3A03 Real Analysis I (taught by David Earn in Term 1 of 2017–18)

The subject matter of this course is the real number system and real-valued functions of real variables. At face value, many of the topics may appear to be the same as those in first year calculus, but the emphasis and goals of the course are completely different. The course focuses on learning to construct rigorous proofs that provide a firm foundation for the kinds of calculations and manipulations learned in elementary calculus courses. We begin by defining numbers and consider what properties characterize the real numbers in particular. We re-examine sequences of real numbers and prove fundamental results such as the Monotone Convergence Theorem (every bounded monotone sequence of real numbers is convergent). We make the notions of limit and continuity completely rigorous, and generalize familiar concepts such as open and closed intervals to include much more complicated sets. In the process, we reveal many subtleties about real numbers and real-valued functions that form a prelude to many fascinating topics in higher mathematics. The course is required for several of the department’s undergraduate programmes, because much advanced mathematics depends critically on a solid understanding of real analysis.

The majority of students taking Math 3A03 have taken Math 2R03, Math 2X03 (or ISCI 2A18) and Math 2XX3. However, none of the specific content of these Level II courses is necessary to study real analysis. Students who have done well in first year calculus and linear algebra can request permission to take Math 3A03 in their second year.

Students will be evaluated using a set of graded assignments, midterm tests, and a final exam.

Math 3B03 Geometry (taught by TBA in Term 1 of 2017–18)

In the Fall term of 2017, Math 3B3 will focus on the geometry of surfaces.

Throughout high school and the first two years of university, geometric concepts/facts that students come across and use are based on Euclidean geometry. These include (a) the shortest path between two points is the line segment joining them, (b) the sum of the angles of a triangle equals 180 degrees, and (c) given a line and an external point, there is exactly one line passing through the given point that does not intersect the given line.

Yet, we have known for centuries that the earth is round (i.e., not flat), and in spherical geometry, (a)-(c) above are all false. What facts are their replacements? In fact, the earth is also not perfectly round. So spherical geometry is not exactly applicable and we need to learn more general tools which allow us to deal with and analyse the geometry of arbitrary surfaces. Besides the surface of the earth, interesting surfaces occur in abundance in nature and by human creation. These include interfaces between liquids and gases, or solids and liquids that arise from physical (e.g., geothermal) processes, the surfaces of body parts and organs, membranes and cell walls, as well as curved portions of buildings and the surfaces of aircrafts and missiles.

A fundamental concept that will be explored in Math 3B3 is the curvature of a surface. A point on a surface has positive curvature if near the point the surface lies on only one side of the tangent plane at the point in question. The point has negative curvature if all sufficiently small pieces of the surface containing the point lie on both sides of the tangent plane. Thus the sphere \( x^2 + y^2 + z^2 = 1 \) has positive curvature everywhere while the saddle \( z = x^2 - y^2 \) has negative curvature near the origin.

For a given surface are we free to bend it (without tearing it) into any shape we want? For example, if we let a little bit of air out of a soccer ball, which starts off as round, we can make some points have negative curvature. But can we bend the soccer ball so that it has negative curvature
everywhere? Can we bend the surface of a doughnut within \( \mathbb{R}^3 \) so that it has zero curvature everywhere?

The answers to these and similar questions will be discussed in Math 3B3. The only prerequisite is a firm grasp of advanced calculus and linear algebra.

**Math 3C03 Mathematical Physics I** (taught by Tom Hurd in term 1 of 2017–18)

As its title implies, this course aims to teach you the fundamental mathematical methods which are essential for solving a wide variety of physical problems. Examples will be taken from quantum mechanics, heat flow, waves, and mechanical coupled systems. We will focus on the theory of differential equations, and especially its relation to linear algebra. By the end of the course you will be able to solve a wide variety of ordinary and partial differential equations from physics and engineering (e.g., heat equation, wave equation, Schrödinger equation). You will find this course especially useful for understanding and solving problems in quantum mechanics, classical mechanics and electro-magnetism.

**Math 3CY3 Cryptography** (not taught in 2017–18)

**Math 3D03 Mathematical Physics II** (taught by Tom Hurd in term 2 of 2017–18)

Math 3D03 Mathematical Physics II, the sequel to Math 3C03, provides a fast moving tour of some mathematical themes that provide completely fresh insight into the equations of physics. The first of the two main themes is the surprising (perhaps even unnatural) usefulness of complex numbers for understanding and untangling physics problems. One might be prepared to believe this on the basis of what one learns in 3C03, for example by noting the similarity between the heat and wave equations, but Math 3D03 gives us a chance to delve deeply into this remarkable fact. The Cauchy Integral formula and Fourier Inversion Theorem are two of the landmark results we will achieve.

The second main theme is to develop a foundation in probability and statistics. Along the way, we will use our new skills in complex analysis to efficiently develop the most important results. For one example, the Central Limit theorem will follow easily from the Fourier Inversion theorem. In addition to theoretical underpinnings, a quick introduction to statistical reasoning will lead us to some practical methods of statistical inference and hypothesis testing.

**Math 3DC3 Discrete Dynamical systems and Chaos** (not taught in 2017–18)

**Math 3E03 Algebra I** (see Math 3GR3)

**Math 3ET3 Mathematics Teaching Placement** (students find their own supervisor)

Math 3ET3 is a math placement course — think of it as a co-op placement in a high school math class, for no less than 60 hours. It can be done in Fall, Winter or Spring/Summer terms (i.e., whenever high school students are not on vacations). You might be asked to observe the teacher, help students in class and outside, prepare lesson plans, deliver some lessons, and whatever else your placement supervisor and you agree upon. It is supposed to be an active and rewarding experience for you.

You need to identify a high school teacher (your placement supervisor) who is willing to take you as a helper. Your placement supervisor evaluates your work, and their evaluation counts for 60% of your grade in 3ET3. Part of the requirement is that you keep a teaching diary.

You also have an academic supervisor, who needs to be a faculty member in Math and Stats department (again, you need to find your supervisor). The academic supervisor makes sure that
there is a theoretical component to your experience. This means that you will be asked to read papers and/or book chapters relevant for your placement, and write three essays/paper critiques. The academic supervisor is responsible for 40% of your 3ET3 grade.

To apply for 3ET3, go to the Science co-op office and ask for an application; the form will tell you what the deadlines are. Once you fill out the form and identify your placement supervisor, discuss course details with your academic supervisor.

Math 3F03 Advanced Differential Equations (taught by Gail Wolkowicz in term 1 of 2017–18)

Mathematical models involving linear and nonlinear differential equations are used in all branches of the natural and social sciences and in engineering. Predictions based on models help us better understand the phenomena under investigation and how to control them and create better designs.

The emphasis of the course will be to develop skills in analyzing systems of linear and nonlinear differential equations using analytical and geometrical approaches widely applicable in science and engineering. For linear systems of differential equations, explicit solutions can be obtained. However, for nonlinear systems this is not usually possible and so the aim is to determine the qualitative behaviour of the solutions, e.g. the existence, uniqueness, and long term behaviour.

The course will focus on studying systems of ordinary differential equations including linear systems, nonlinear autonomous systems in the plane, phase portraits, local and global stability of invariant sets such as equilibria and periodic solutions, the Poincaré-Bendixson Theorem, the Dulac criterion, and Lyapunov’s method for higher order nonlinear systems.

Students will be expected to learn precise statements of Definitions and Theorems and to use them appropriately (i.e., to verify the hypotheses to determine when they apply and when they do not, and make appropriate conclusions). Students will be introduced to software for time-series simulations, phase portraits, and bifurcation diagrams.

Math 3FF3 Partial Differential Equations (taught by Walter Craig in term 2 of 2017–18)

This course introduces the student to the study of Partial Differentiation Equations primarily through an in depth examination of the simplest and most important PDE’s that arise in the sciences: the wave equation, the heat equation, and Laplace’s equation. Each of these classical equations is considered for the most part in the plane, where the differences in the underlying structure of solutions are most clearly exposed. The interplay between the mathematical solutions to these equations and their physical interpretations plays a crucial role in the development of the ‘well-posedness’ of the theory. The principle of causality in the wave equation, the maximum principle and principle of decreasing energy in the heat equation, and the mean value theorem in Laplace’s equation are just a few important examples of this symbiotic relationship that contributes to making the solving of PDE’s a most exciting and rewarding application of mathematics to the sciences.

Linear algebra and calculus both play a critical role in solving PDE’s and are especially prevalent in the early stages of the course when more general PDE’s are briefly considered.

Math 3FM3 Mathematics of Finance (taught by Traian Pirvu in term 1 of 2017–18)

Math 3FM3 is the perfect place to start learning about the rigorous mathematical formalism underpinning the modern theory of Mathematical Finance. In this class, we will focus on financial models in discrete time and learn about options and forwards, efficient markets, no arbitrage pricing, the binomial asset pricing model, portfolio strategies, stochastic processes, conditional expectation, martingales, optimal portfolio selection, and exotic options. After this course, you will have a clear
understanding of the key concepts of modern Mathematical Finance and you will be exceptionally well prepared to embark onto the next stage — the continuous time modeling paradigm taught in the Math 4FM3 course. Together, 3FM3 and 4FM3 prepare students for a career working in a research department of a financial institution developing and working with models of financial risk. They also cover the material of the Society of Actuaries MFE exam.

Math 3G03 Problem Solving (not taught in 2017–18)

Math 3GP3 Geometric Ideas in Physics (not taught in 2017–18)

Math 3GR3 Abstract Algebra (taught by Adam Van Tuyl in term 1 of 2017–18)

The goal of this course is to introduce the fundamental objects of abstract algebra: rings and groups. Rings and groups are important objects that appear in many branches of mathematics. Group theory has its roots in the study of roots of polynomials and plays a central role in the study of symmetry. Ring theory has some of its origins in the study of prime numbers, and plays a pivotal role in current areas of research like algebraic geometry.

In this course we will illustrate the definitions of groups and ring with numerous examples (the set of all integers has both a ring structure and a group structure). By the end of the course, you will be introduced to some of the standard terminology to related to groups and rings (e.g., subgroups, subrings, group and ring homomorphisms, quotient groups and rings), and be introduced to important families of groups and rings (e.g., symmetric and alternating groups, integral domains, and fields).

Another aim of this course will be to explore and understand various proof techniques. The prerequisite for this course is MATH 2R03. Students will be evaluated using a set of graded assignments, midterm tests, and a final exam.

Math 3H03 Number Theory (not taught in 2017–18)

Math 3MB3 Introduction to Modelling (taught by Ben Bolker in term 1 of 2017–18)

Introduction to mathematical modelling using analytical and numerical methods with a strong focus on physical applications. The goals of the class are fourfold. First, to give an overview of how to construct and understand models involving differential equations in a variety of fields such as biology, medicine, chemistry, physics, economics and other areas of natural and social sciences. Second, to analyze these models analytically, using algebra and calculus. All necessary mathematical concepts will be introduced or reviewed. Third, to make use of software such as MATLAB and R to perform computational analysis of these models. Last, to understand how to interpret model results back into the original motivating context.

Math 3NA3 Numerical Linear Algebra (taught by Nicholas Kevlahan in term 1 of 2017–18)

This course is a part of the computational mathematics sequence which also involves MATH 2T03. In MATH 3Q03 we will be studying key questions of numerical analysis such as approximation of functions and approximate differentiation and integration. We will therefore see how various problems arising in calculus (both in single and in multiple variables) can be solved approximately, but with controlled accuracy, using computer algorithms. In addition to proving theorems about various numerical methods, we will develop, analyze and implement actual computational algorithms using MATLAB. We will also show how computational techniques can be used to illustrate and verify different results of mathematical analysis. As a highlight of the course, we will introduce Chebfun (http://www.chebfun.org/) which is a MATLAB toolbox for performing hybrid numerical-symbolic computations with very high accuracy.
Math 3Q03 Numerical Explorations (see Math 3NA3)

Math 3QC3 Introduction to Quantum Computing (not taught in 2017–18)

Math 3T03 Inquiry in Topology (taught by TBA in term 2 of 2017–18)

Topology comis the field of mathematics concerned with studying a formalization of the notion of shape. In topology, we are interested in properties of spaces which remain unchanged under continuous deformation by way of stretching and bending. Hence the joke about a topologist being someone who cannot tell the difference between a coffee cup and a doughnut.

In this course, we will define the simple and abstract axioms of topology and basic definitions associated to them (such as continuity of maps, connectedness and compactness). We will also see how topology can provide beautiful insight into practically all other branches of mathematics including analysis, algebra, graph theory, functional analysis. This course will also expose the students to both mathematical rigor and abstraction, giving them an opportunity to further develop their mathematical knowledge.

Topics include topological spaces, metric spaces, continuity, homeomorphisms, connectivity, compactness, and separation axioms.

Math 3TP3 Truth and Provability (taught by Bradd Hart in term 2 of 2017–18)

Math 3U03 Combinatorics (not taught in 2017–18)

Math 3V03 Graph Theory (taught by TBA in term 2 in 2017–18)

In this course we will examine the basics of graph theory. Roughly speaking, a graph consists of two parts: vertices or nodes, and edges that connect vertices. There are many applications of graphs, such as scheduling or modelling computer networks and social networks. This course will introduce the basic terminology associated with graphs, look at special families of graphs (e.g., Euler or Hamilton graphs), and discuss colouring problems, among other things. The in-class lectures and exercises will place an emphasis on proving results about graphs.

Math 3X03 Complex Analysis (taught by Eric Sawyer in term 2 of 2017–18)

In this course you will learn the fundamental ideas and surprising results in the study of functions of a complex variable. Once a complex function is differentiable it satisfies a whole range of magical properties: surprising identities relating its integrals and derivatives; you can count its zeros by calculating an integral; the level curves form orthogonal webs; and you can turn the plane into the sphere. By opening up to complex values, you will solve problems involving real variables and functions, which seemed impossible from the real number point of view. Topics include analytic functions, series, residue theorem, conformal maps with applications.

Math 3Z03 History of Mathematics (taught by M. Lovric in term 2 of 2017–18)

Mathematics is an ancient subject and its history stretches back to the dawn of human civilisation. Why do we measure time in units of 60? We are so busy with our day-to-day concerns about the current mathematical curriculum that we rarely have time to look back and appreciate our discipline as an integral part of human culture and to appreciate the beauty of mathematics—not to mention all the immense scientific and technological advances that are based on mathematical understandings and interpretations of the Laws of Nature.

Math 3Z03 is a student-centred inquiry course, which will help students in Mathematics and Statistics to bridge this fundamental gap in their historical understanding of the subject. Students will be studying mainly on their own (of course under the watchful guidance of the instructor!)
about specific interesting topics in the rich and vast history of mathematics, and will present their findings to other students. The students will then write short but informative reports on what they presented in class, so both oral and written communication skills are honed! In addition, there will be a number of regular assignments consisting of somewhat difficult but very interesting well-known historical problems, and in-class quizzes and (possibly) tests to check your comprehension of the presentations and of the assignments.

**Stats 3A03 Applied Regression Analysis with SAS** (taught by Angelo Canty in term 1 of 2017–18)

One of the most common uses of statistics is modelling the relationship between a response variable and one or more explanatory variables. The simplest form of modelling is linear regression which will be taught in this course. The course will start with simple linear regression in which the relationship between mean response and a single explanatory variable is a line. The course will examine the derivation and properties of least squares estimators of the intercept and slope. Extensions to multiple explanatory variables as well as categorical explanatory variables will also be covered. All statistical modelling involves some assumptions and diagnostics are available to check these assumptions in regression. More advanced aspects of regression which can be applied when one or more assumption does not hold will also be covered, as will selection of a good set of explanatory variables to use when there are many such variables available in a dataset. Many practical examples will be given using the SAS statistical analysis software. In parallel with the lectures, there will be weekly computer labs in which the students will learn the basics of SAS and its use for regression.

**Stats 3D03 Mathematical Statistics** (taught by Narayanaswami Balakrishnan in term 1 of 2017–18)

The course’s main objective is to provide the students with a more formal treatment of the concepts and methods used in statistical modeling and analysis of data. It is a required course for most statistics programs across the country. In contrast with the approach taken in STATS 2MB3, STATS 3D03 delves with more vigor into the derivation of the methods and their properties.

The course starts with a review of the axioms of probability. This includes the classical Bayes Theorem. Discrete and continuous random variables is the next topic, with emphasis on calculation of expected values, transformations and the introduction of the highly useful moment generating function (MGF) and its properties. The MGF plays a sustained role throughout the course as an effective tool that facilitates the derivation of distributions of key statistics, including parameter estimators. A good portion of time is dedicated to the study of multivariate distributions with particular emphasis on the bivariate case. They present a unique opportunity for the use of basic multivariate calculus.

The following topics focus on inferential methods, including the derivation of confidence intervals using pivotal quantities from different distributions. Attention then is given to statistical tests and their properties. Judgement is developed to distinguish efficient methods from others that show reduced power to achieve inferential objectives.

**Stats 3G03 and 3H03 Actuarial Mathematics I, II** (taught by Margaret Verbeek in terms 1 and 2 of 2017–18)

Are you considering a career as an actuary? Would you like to dip your toes into an introductory course or acquire more in-depth knowledge of the profession? Stats 3G03 and 3H03 will introduce you to the world of life insurance and the blend of mathematics, statistics and business knowledge that will be used to manage financial risks.
Stats 3G03 begins with the introduction of common life insurance products, discussion of survival functions and life tables and the development of life contingent payment models that will be used to assess the costs associated with these products.

Stats 3H03 takes those concepts and introduces more complex situations: sophisticated products, multiple insured lives, multiple decrements, and a more complex investment environment. The life contingent payment models discussed in Stats 3G03 will be amended to reflect these complexities and the student will learn about methods used to fund these models.

Completion of Stats 3G03 and 3H03 will achieve the Society of Actuaries objectives of developing the students knowledge of the theoretical basis of contingent payment models and the application of those models to insurance and other financial risks. If you decide to pursue a career as an actuary, the combined Stats 3G03 and 3H03 courses will prepare you for the Models for Life Contingencies professional exam offered by the Canadian Institute of Actuaries/Society of Actuaries.

Stats 3PG3 Probability and Games of Chance (taught by Fred Hoppe in term 2 of 2017–18)

Stats 3S03 Survey Sampling (taught by Roman Viveros in term 1 of 2017–18)

Survey sampling is a collection of principles and procedures involved in the planning and conduction of a sample survey from a finite population, and in the corresponding data analysis. When conducting a survey, the main objective usually is to accurately estimate a population quantity such as a population average, total or proportion. Emphasis is placed in the course on the merits, appropriateness and properties of a variety of well-known probability sampling schemes for a finite population, and on the associated statistical analyses. While simple random sampling with and without replacement are the traditional methods of sampling, other sampling designs such as systematic, stratified and cluster designs may be easier and cheaper to implement. The properties and associated estimation methods for all of these sampling designs will be discussed in detail. The topics will be presented with reference to sample surveys from a variety of applications. The course provides valuable training for someone interested in working for Statistics Canada and other government agencies, polling companies or in marketing research.

The course prerequisites are STATS 2D03 and one of ARTS&SCI 2R03, STATS 2MB3.

Stats 3U03 Stochastic Processes (not taught in 2017–18)