Abstract

As part of a second year introductory probability and statistics course (Arts and Science 2R03), Problem Based Learning exercises were introduced in tutorials to front-run course material prior to exposure in lecture. We hypothesized that when compared to exposure by lecture and traditional tutorials, exposure to material by PBL exercises would lead to better learning and retention of material. We began by measuring the Fall 2010 cohort of A&S 2R03 for a baseline level of retention with respect to one particular topic studied in A&S 2R03. During the Winter 2012 term, PBL exercises were used in weekly tutorials with the intention of developing some preliminary understanding of select course topics prior to lectures. One year following the completion of the course, students from A&S 2R03, Winter 2012 will be measured for retention of material studied in A&S 2R03, which will be compared against retention results from the Fall 2010 cohort.

Introduction

Problem Based Learning is a “learner centered” pedagogical method which originated in McMaster University’s School of Medicine and which has taken many forms since its inception (Savery 2006, 9-10). Curriculum inspired by the PBL philosophy exists in many forms in all subject areas and at all levels of formal education. At its broadest level, PBL consists of students working in small groups to solve complex problems, with the intent being to have students discover a particular procedure for solving a problem or to develop some intuition behind a concept. With respect to our study, PBL exercises served the latter purpose, intending to promote some preliminary understanding of statistical concepts prior to lectures. The intent behind introducing PBL to the course was to have students approach the learning of probability and statistics intuitively, rather than by rote memorization of formulas and doing plug-and-chug style questions, which we believe leads to poor retention.

Olson et Al. (2011) distinguish between PBL problems and traditional mathematics exercises thusly:

“[e]xercises require students to use a strategy repeatedly, and assume that the student will gain understanding of mathematical ideas through repetition. Problems require students to apply their knowledge by interpreting the problem context, gathering needed information, identifying possible solutions, evaluating options, and presenting conclusions” (734).

PBL problems can take one of (at least) two forms: highly scaffolded problems which guide students through a clearly defined sequence of steps in developing a particular process or
formula for solving a problem; or highly un-scaffolded, vaguely worded problems which require students to interpret the problem and develop some intermediate steps to solve. Both types of problems would avoid providing exactly the information required to answer the question. The first type of problem would be particularly well suited to having students develop some intuition behind statistical concepts, while the second would be well suited to promoting the development of problem solving skills. Both types of problems were used during the course of our study, in addition to some more straightforward problems requiring application of material learned in lectures. In using PBL exercises for tutorials, our goal is to have students engage in active problem solving during tutorials rather than the passive listening that is characteristic of traditional tutorials.

By design, PBL problems generally draw from real life examples, and in mathematics classes PBL has often been used to combine the study of mathematics with that of biology, health sciences, and other fields in the social and natural sciences. The interdisciplinary nature of PBL problems has been touted as a strength of the PBL model (Savery 2006, 10).

In a PBL setting, the role of the instructor differs from that in a traditional lecture setting. Rather than delivering material by lecturing, the instructor “ guides the learning process and conducts a thorough debriefing at the conclusion of the learning experience” (Savery). During the course of the PBL session, the instructor serves as a “model learner”, “ask[ing] students the kinds of questions that they should be asking themselves to better understand and manage the problem” (Clarke et al. 2004, 8; Barrows). The instructor divulges as little information as possible in response to student questions, only clearing up any misconceptions as necessary and requiring the students to discover the relevant steps on their own. During the debriefing session, the instructor solicits input from the various groups about the approaches they took to the problem, asking the students to justify each step of their solutions, and by doing so constructs a correct solution using student input. The various sources of input during take up of the exercise may also illustrate different correct approaches to solving the problem. The instructor concludes by identifying the important points illustrated by the problems that the students are intended to take away.

Proponents of PBL identify several benefits to the PBL model in addition to the obvious benefit of improving problem solving skills. Dods (2001) posits that “PBL engages students as active learners, promotes habits of mind that support higher order thinking, and is an effective vehicle in conveying factual knowledge” (434). Further, Duch et al. (2001) argue that “students who learn concepts in the context in which they will be used are more likely to retain that knowledge and apply it appropriately”. This improvement in retention is precisely what Dr. Lozinski hoped to accomplish by introducing PBL to A&S 2R03 tutorials. The take up of PBL problems places emphasis on communicating the significance of answers, and so the PBL model
of learning might also promote improved communication skills. Barrows (1996) also notes that in using PBL exercises the instructor receives continuous feedback on student learning and performance, and can more effectively intervene in areas of difficulty.

Conversely, teaching by PBL takes substantially longer than delivery of material by traditional lecture methods, so using PBL as the sole method of delivering course material is not a viable option unless the instructor is willing to make significant sacrifices in the amount of content covered. Some observers also argue that students require multiple exposures (i.e., more than one or two courses) in order to gain the full benefits of PBL. Further, PBL demands a high level of educational maturity and ability for self-directed learning from students, and thus may not necessarily be suitable for every course and audience. Lieux (2001) notes that less academically mature students will tend to prefer an instructor-centered course in which expectations are clearly defined and course problems resemble the “exercises” described by Olson at Al. above. We felt that PBL would be particularly successful as a method of instruction for Arts and Science students, who tend to be high achieving and motivated students. Arts and Science students also have a high degree of familiarity with one another, which we felt would be conducive to small group work. Finally, as we will see, there have been no consistent findings in pedagogical research demonstrating the superiority of PBL in promoting learning or retention.

Arts and Science 2R03 is a one semester introductory probability and statistics course intended for second year students in the Arts and Science program. The 2010–2011 academic year was the first time that this course was a one semester course, as well as the first year that Dr. Lozinski taught the course. Arts and Science students take a mandatory full year calculus course in first year. For the vast majority of Arts and Science students, this is the only background they have in mathematics or statistics at the university level prior to A&S 2R03.

**Background and Supporting Research**

We began with a review of literature on PBL in mathematics, statistics and science education, focusing in particular on literature dealing with mathematics courses or courses at the university level. The literature associates a variety of benefits to the PBL model, but does not conclusively point to PBL as a successful model for improving learning or retention, sometimes even providing contradictory conclusions. It is worth repeating that PBL is not a uniquely defined pedagogical method, and different interpretations and approaches to PBL may account for the variance in success. The following is an overview of several of the most relevant findings.
Clarke et Al. (2004) evaluated the effects of a PBL initiative in three grade 10 mathematics classes in California high schools. They felt that the success of PBL would be more adequately measured by the changes in student perceptions about the importance and usefulness of mathematics, because these perceptions are likely to influence whether students subsequently continue to study mathematics (7). They argue that

[even] if it could be demonstrated that student achievement [in mathematics] was enhanced by a problem-based program, ... the ultimate value of such a program would depend still on whether the student chooses to continue to study mathematics, develops a set of beliefs which supports and empowers further learning, and sees any relevance in the skills acquired in class (7).

They compared student perceptions about mathematics from students in the PBL course to a control group of students studying math at the same grade level in a non-PBL setting. They asked students to describe their perceptions using a variety of adjectives towards mathematics and quantified student responses by assigning a value of +1 to positive adjectives and a value of -1 to negative adjectives, and summing the results for each student (11). They concluded that students from the PBL treatment “held a significantly more positive attitude towards their mathematics classes” than students from the control group (14). They found that students in the PBL course were more likely to perceive mathematics as being relevant to everyday tasks such as planning a vacation or painting a house (14). They found students from the PBL classes were more likely to find value in discussing ideas with other students (13). Conversely, they noted that control group students were more likely than PBL students to view “drill and practice” as the best way to learn mathematics (13). All of these differences were found to be statistically significant. Finally, they noted that students from the PBL classes scored marginally better on the SAT mathematics test administered in the following school year, though for only one of the classes was this result found to be statistically significant (10).

For a grade 12 biochemistry course at an Illinois high school, Dods (2001) delivered content in one of three ways: PBL; traditional lecture; and a mixed PBL and lecture format (423). Dods was interested in looking at whether retention of course content would be promoted by PBL. He began by pretesting students for a baseline level of knowledge with respect to a wide range of biochemistry terms and concepts, measure depth of understanding of concepts on a 0-3 scale, with a score of 0 indicating no understanding of the concept and a score of 3 indicating an “excellent” understanding of the concept (427). He noted that “improvement in understanding of [biochemistry concepts] was independent of the pedagogy in which the item was encountered (lecture, PBL, or lecture and PBL)”, indicating that in this instance PBL was no more effective than traditional lectures in promoting learning (429). Approximately three weeks following the initial exposure to a given concept, students were
tested for retention (433). It was found that “retention of content encountered in a PBL environment was significantly greater than when encountered in a traditional lecture environment” (433). Furthermore, it was found that content items that demonstrated improved understanding on the retention test from an earlier test were more likely to have been encountered in a PBL setting (433). Student course evaluations indicated that students felt that the course covered less content than would have been covered had the course been delivered entirely in lecture format, but that content delivered in PBL format was better understood (433).

Olson et Al. (2011) studied the effects of class size and teaching approaches on student achievement in a first year university precalculus course. There were four course sections of the precalculus course, each with a different teaching approach or class size: a traditional large-class lecture course which served as a control group; a traditional lecture course with a reduced class size; a class which used traditional lectures for three of the weekly classes and which allowed students opportunities to work in small groups during the fourth weekly meeting; and finally a class which placed an emphasis on problem based learning (732). The difference in approaches was not advertised prior to the course, so enrollment across the different sections was assumed to be random (736). In the PBL treatment, the instructor “continually asked students to solve problems and reflect on the strategies used” throughout the course (739). New topics were introduced by posing questions, and students were given a few minutes to explore the questions, after which the instructor would solicit student input (739).

The various approaches were evaluated using test scores on common mid-term examinations and a voluntary survey intended to measure “students’ levels of anxiety and beliefs about the importance of mathematics” (739). For students who continued to study calculus in the following semester, the researchers also looked at their performance in calculus (740). Students in the PBL section scored higher than other students on all four of the semester’s mid-term examinations as well as the final examination, with the differences most pronounced in later examinations (740). This result was found to be statistically significant (741). There were no significant differences found between the other three groups (741). Further, teaching approach and class size was found to have a statistically significant effect on overall course grades, with a greater proportion of PBL students earning an overall grade of A or B (742). No significant differences were found with respect to students’ anxiety levels or beliefs in the importance towards mathematics (743-744).

Olson et Al.’s findings were most surprising with respect to the students who continued to study calculus in the following semester. Teaching approaches were not found to have significantly influenced whether students continued on to study calculus (745). However, the researchers found that students from the PBL course were no more likely to succeed in calculus
than students from the control group ("success" being defined as earning a grade of C or better) (744-745). Conversely, a significantly larger proportion of students from the small class size and small group learning sections were found to have “succeeded” in calculus, with over 90 percent of students deemed to have been “successful” in each of these sections compared to 70 and 68 percent in the PBL and control sections respectively (745-746). This finding suggests that the benefits of PBL did not continue with students beyond the course in which it was used. We contacted one of the researchers to discuss this surprising finding, and she conjectured that the use of PBL has to be sustained across multiple courses to retain its benefits.

Metz (2008) conducted a study looking at the effects of introducing statistical concepts using PBL to a first year university-level biology course taken by students majoring in the biomedical sciences (318). Statistical content was delivered as part of lab exercises, and involved small group work on biology related problems (319). Some students entered the course having taken statistics previously; some took statistics concurrently, and differences were noted between these groups and students who had no experience with statistics at all (319). Students were tested both at beginning and at the end of the course using a combination multiple choice/short answer test (319). It was found that the PBL activities increased statistical knowledge for all students as measured by “normalized gain” (increase in scores pre-/post-treatment divided by possible increase in scores), with increases unsurprisingly the most pronounced among students taking statistics concurrently (320-321). This result was found to be statistically significant (320). Students enrolled in this course were retested one year following the completion of the introductory biology course for retention of statistical knowledge (319). Students were found to have “retained their statistics knowledge at or above where they were after the completion of introductory biology” (324).

Smith (2006) and colleagues conducted a study looking at the effects of a PBL on students’ proof-writing abilities in an introductory university-level number theory course. The researchers were concerned that students who simply watched instructors construct proofs in lectures perceived “the construction and writing of a proof as an algorithm to follow rather than a creative process for solving a problem” (73). The course was taught in three sections, with two using a PBL approach and the other using a traditional lecture-based approach (77). Students in the PBL sections were given a list of theorems to be proven every few weeks, which were “designed to guide students through a logical sequence of mathematical propositions and concepts” (77). Students were instructed to work on these proofs outside of class and students would be asked to write out their proofs on the board and explain them at the beginning of each class (77). The instructors would then facilitate a discussion about the presented proofs, expecting the class to come to a consensus about the correctness of the proofs (77).
The researchers conducted observational interviews with students from both the lecture and the PBL-oriented sections of the course in which students were asked to construct proofs for several mathematical statements and asked to describe their thought processes as they were doing so (81). These observational interviews were only conducted with three students from the PBL group and two students from the control group, but several differences were noted between the approaches taken to proofs by students from the PBL class and the control group. First, students from the control section began a proof by “throwing strategies at the problem in hope of finding something that would work”, for instance, by enumerating proof strategies (contradiction, induction, etc.), trying to recall theorems that might be relevant to the proof, or trying to recall similar problems that they had seen before (81). The researchers interpreted this finding as a tendency to simply search for the quickest way to find a proof, rather than making a real attempt to understand why the statement is true. Conversely, students from the PBL group began the process of constructing a proof by trying to make sense of and rewriting the statement in their own notation before making any efforts at a proof, which the researchers identify as a crucial (and desirable) difference (82). PBL group students were also found to be more likely to use concrete examples in an attempt to find patterns and understand why the statement was true (81, 87). One control group student was quoted as indicating that a proof was finished when the desired conclusion is obtained, which the researchers interpreted as a propensity to view proof writing as a procedural exercise (81), whereas students from the PBL section found to place a higher emphasis on expressing their proofs clearly and making sense of the mathematical concepts behind the proof (87).

We also looked at studies by Shore et Al., Coleman, and Lieux. The following is a brief highlight their relevant findings. Shore et Al. (2004) used PBL to bridge the study of mathematics and the health sciences for students in a college level health sciences program, incorporating examples dealing with anatomy and other health science subjects into the mathematics curriculum. Students in PBL were “consistently” found to have “scored significantly higher” than a control group of students on a common test, though this difference in performance was not quantified (183, 187). Coleman (1998) studied the effects of PBL on teaching photosynthesis to a group of fourth and fifth grade students (387). The students were tested and classified as having an “average” or “high” intentional approach to learning, and a group of students identified as having an “average” intentional approach to learning was taught using a scaffolded, collaborative PBL methodology (387). Later test results showed that the “average” intentional students who learned in a PBL setting performed on par with students determined to have a “high” intentional approach to learning and performed better than students in an “average” intentional control group (400). These results were found to be statistically significant (400). Lieux (2001) measured differences in self-reported experiences in a food sciences course between students taught in PBL and traditional lecture format. Students in the PBL treatment self reported being “stimulat[ed] to high intellectual effort”, a higher level
of engagement with course material, and higher involvement in discussion and answering of own questions. Attendance rates were found to be 25-35% higher for the PBL class. However, the researcher found no significant differences in performance between the two sections on common examinations.

**Procedures and Implementation**

We measured the Fall 2010 class of Arts and Science 2R03 for retention of course material by asking students to do an anonymous, one question survey. A copy of the survey question is provided in Appendix A. Students were not alerted to the nature of the survey question beforehand, so we anticipate that students came in with no preparation. The question was one that was identical in structure to a final exam question from A&S 2R03, Fall 2010. The survey responses were coded with an alphanumeric code and paired with a photocopy of the matching student’s original final exam response. The survey question dealt with the topic of conditional probability, and students could use Bayes’ Formula to answer the question. However, the question could also be solved intuitively, without the use of any formulas. The topic of the retention question was selected specifically for this reason. We assumed performance on this survey question to be a baseline measure of retention, and the difference between the performance on the survey question and the original test question to be a measure of decay of knowledge one year removed from the completion of the course. Students were also asked whether they took any additional mathematics or statistics courses subsequently which may have affected their performance on the retention question. In addition, we noted whether students attempted to use a formula to answer the question (and whether they successfully did so), whether they showed an understanding of the concept of conditional probability, and whether they demonstrated sound mathematical reasoning, each on a numerical scale of 0-3.

The Winter 2012 class of A&S 2R03 consisted of 73 students. The vast majority of the students were second year Arts and Science students, though a few were in their third or fourth year of the Arts and Science program. The students were assumed to be distributed more or less randomly and evenly across the three tutorial sections, with between 20 and 25 students in each section. There were 11 weeks of tutorial sessions – the first was an introduction to Minitab statistical software, and the remaining 10 were dedicated to PBL exercises. Tutorials took place in a computer lab so that students could consult internet sources if necessary. Tutorials occurred on Friday, Monday and Wednesday mornings, and lectures occurred on Monday, Wednesday and Thursday afternoons. Each tutorial cycle began with Friday tutorials, and the tutorial activities were designed so that Friday and Monday tutorial exercises would front-run material from Monday lectures. We were interested in seeing if there would be any
difference in effectiveness of the PBL exercises between the Friday and Monday tutorial sessions which preceded the lecture and the Wednesday tutorials where students would have the benefit of material from Monday lectures before doing the PBL exercises.

Three senior Arts and Science students, who had previously taken the Arts and Science statistics course but who have not studied statistics extensively beyond the course, each served as teaching assistants for one of the three weekly tutorial sessions. In addition, the student researcher attended all but two of the PBL tutorial sessions, observing and assisting as necessary. On the Tuesdays prior to each tutorial cycle, the student researcher and the three teaching assistants would work out solutions to the week’s problem as a small group in PBL fashion. In doing so, we could gauge the approximate amount of time that the problems might take and anticipate potential difficulties that students might encounter while working through the problems. At the conclusion of these sessions, the intended takeaways for the week’s tutorial problems were provided by Dr. Lozinski.

In tutorials, students worked in groups of four or five, which were randomly selected and changed every week. The PBL exercises were intended to take approximately half of the tutorial sessions, with the remainder of the tutorials devoted to traditional worked examples delivered by the teaching assistant or addressing student questions as necessary. However, the PBL portion of the tutorials tended to run longer than the allotted time due to the length of the problems, the necessity of a thorough debriefing, and a lack of student questions for the majority of tutorials. Feedback was solicited on the tutorial exercises after three weeks and subsequent exercises were calibrated for length, difficulty, and content as necessary. Among the topics dealt with in PBL exercises were the normal distribution, conditional probability, confidence intervals, and hypothesis testing. Students were expected only to engage in problem solving during the PBL sessions, and no expectations were placed on the students with respect to performance. These expectations were communicated clearly at the beginning of the semester, and tutorial problems were marked for completion only. All of the problems were created by Dr. Lozinski. Two of the problems that were used during the semester are provided in Appendix C for illustrative purposes. Formal solutions to these problems were provided to individual students only upon request. Feedback on the tutorial exercises was solicited once again at the end of the semester to gauge changes in perceptions about the usefulness and success of the PBL exercises.

The final examination for A&S 2R03, Winter 2012 will contain one question that deals with conditional probability which students may use Bayes’ Theorem to solve. The question will be identical to the one that students from the A&S 2R03, Fall 2012 saw on their final exam. One year following the completion of A&S 2R03, Winter 2012, students from the course will be asked to anonymously complete a survey question dealing with conditional probability in an
identical fashion to students from A&S 2R03, Fall 2010. Once again, we will use performance on the survey question as a measure of retention and the difference between performance on the survey question and the original exam question as a measure of decay of knowledge. We will also note whether there are any differences in tendencies with respect to use of formulas. These results will be compared against retention scores from the Fall 2010 cohort of A&S 2R03.

**Hypothesis**

We hypothesize that when compared to exposure by lecture and traditional tutorials, exposure to material by PBL exercises and lecture will promote better retention of course materials one year after the completion of the course by appealing to higher order levels of cognition.

**Results and Discussion**

We received 22 responses to the retention question from the Fall 2010 group. These responses were graded by Dr. Lozinski on a 1-5 scale, with distinctions made between various levels of 4 and 5 scores. In addition, judgments were made about whether students used a formula to answer the problem (and whether they used the correct formula), whether they demonstrated an understanding of conditional probability, and whether they used sound mathematical reasoning, each on a 0-3 scale. A chart of the results has been provided in Appendix B. The mean score on the retention question was 3.1773 out of 5 with a standard deviation of 1.8379 among the 22 respondents, compared to a mean score of 4.7636 and a standard deviation of 0.7248 among the same 22 respondents on the original test question. Only one student was judged to have used a reasonable representation of the formula on the retention question, compared to 15 of 22 students relying on the formula to answer the original exam question. Two students scored higher on the retention question than on the original test question.

The scores are displayed in a scatterplot in Figure 1. Original exam scores are represented in the x-coordinate, while retention scores are represented in the y-coordinate. The red dots indicate the 45 degree line, i.e. where original and retention scores are the same, indicating no decrease or improvement in performance. Points above the red dotted ‘line’ indicate improvement from the original test score, while points below the line indicate a decrease in performance from the final exam. The green square denotes the one student who successfully reproduced the relevant formula on the retention question, and we note that this was the only student who performed at exactly the same level on both the original and
retention questions. We anticipate that there was a slight self-selection bias favouring students who performed well in A&S 2R03.

Figure 1: Scatterplot of original exam and retention scores

We found two prevailing tendencies on the responses to the retention survey question. As noted, only one student produced a reasonable representation of the correct formula. Other students who attempted to use a formula to answer the question either used an incorrect formula or did not successfully recall the formula. These students produced among the worst scores on the retention question. Students who successfully answered the retention question tended instead to score highly in the reasoning and understanding of concept categories. Apart from demonstrating poor retention of course material, this suggested to us that students will naturally tend to approach probability problems using their intuition and problem solving skills. Since we cannot rely on students to retain formulas, we feel that it would be more beneficial to
encourage the development of good problem solving skills and teach statistical concepts using an intuitive approach rather than by encouraging the use and memorization of formulas.

Three weeks into the semester, we asked students for feedback on the aspects of the PBL exercises that were working well and suggestions for improvements. Feedback was collected using anonymous surveys. Many comments from the survey suggested to us that the students understood the intent behind the PBL exercises. Many students enjoyed the challenge of the problems and found that being able to bounce ideas off one another was beneficial. Students appreciated the informal, low-pressure environment of the PBL sessions. Some students reported that having some pre-existing knowledge of the concepts going into lecture helped them follow the lectures more easily.

However, students also expressed some frustration over having the wrong idea throughout the PBL sessions. With this in mind, a concerted effort was made to have the facilitators circulate among the groups and discuss the problems with each group at least once during each of the PBL sessions, asking students to explain their approaches to the problem and clearing up any misconceptions as necessary. Students also indicated the first few problems were too long, leaving insufficient time for a thorough take up. As a result, subsequent exercises were designed to be shorter. A few students expressed a preference against the use of problems to front-run material from class, preferring to learn relevant concepts prior to attempting the problems.

Student comments from the end of semester survey indicated that the balance between time devoted to PBL exercises and traditional tutorial activities was improved somewhat later in the semester and that a thorough takeup of the problems was achieved more often, but that a fair balance was still not met. Students also indicated the teaching assistants were more successful in giving feedback immediately, circumventing any egregious errors. The student researcher also sensed an increasing comfort level with problem based learning as the semester went on.

Next Steps, Recommendations and Discussion

In April 2013, we will ask students from A&S 2R03, Winter 2012 to complete a one question survey to measure their retention of course material. Similar to the survey for the Fall 2010 group, marks from the students’ retention question will be compared against scores from the final examinations, and the use of formulas, mathematical reasoning, and understanding of conditional probability will be noted. We will use overall performance on the retention question, student tendencies in using formulas, and ability to demonstrate sound mathematical reasoning as areas for comparison between the two groups.
The weekly PBL problems will continue to be calibrated for difficulty, length, and content for future iterations of A&S 2R03. We felt that it took a few tutorials for students to become comfortable with PBL. To address this, Dr. Lozinski plans to begin the first tutorial cycle with an introductory problem solving exercise, possibly even one that does not deal with probability or statistics, in order to accelerate familiarity with PBL-style exercises. We would also like to place more emphasis on having students communicate the significance of numerical answers. Two of the PBL exercises dealt with the topic of hypothesis testing, and though students were able to master the mechanics of computing a test statistic, there were often questions in tutorial about what the test statistic meant. Dr. Lozinski plans to address this by increasing the number of problems which explicitly ask students to explain in words the significance of their answers.

We are interested in finding ways to deliver a more uniform experience across the three tutorial sections in future iterations of the course. It is the student researcher’s opinion that the three tutorials delivered vastly different experiences. The differences in levels of knowledge and ability to explain statistical concepts among the three teaching assistants will naturally create a different experience across the three tutorial sections, but we would like to find ways to train and prepare the teaching assistants to mitigate these differences.

As mentioned, we also felt that Wednesday tutorials did not achieve the intent of front-running course material, and thus may not have been as effective. Especially in later tutorials, students in the Wednesday tutorial section were able to complete the PBL exercises with relatively little difficulty, with students very frequently consulting their notes from class to answer the PBL questions. Dr. Lozinski plans to address this either by requesting that tutorials be placed closer together, or by assigning a more challenging exercise to students in the Wednesday tutorial section.

As mentioned, we used a variety of types of problems in the PBL exercises. Exercises from early in the semester tended to be of the highly scaffolded variety, while later exercises tended to be of the vaguely worded variety which required students to develop intermediate steps or compute additional information to solve. This was intentional, as we felt that highly scaffolded problems would be friendlier to students experiencing PBL for the first time. A few of the problems were fairly straightforward applications of statistical procedures. We received comments from different students indicating preference for each type of problem. The student researcher would be interested in studying which type of problem was most effective in promoting learning.

Inspired by Dods’ and Metz’s studies, the student researcher would have liked to pre-test for initial knowledge of statistical concepts. The pre-test would consist of A&S 2R03 test-style questions dealing with concepts from A&S 2R03, as well as some distractor questions so as
to not alert students to upcoming topics. In doing so we could also get a measure of students’ statistical problem solving abilities entering the course. We would then measure learning during the course as improvement in performance on the test questions. The student researcher would be interested to see to what extent (if at all) retention scores represent an improvement from pre-test scores.

Finally, as previously noted, practitioners of PBL believe that multiple exposures are necessary to reap the full benefits of the PBL model. There may be little or no benefit to students in the long run if the PBL aspect of A&S 2R03 is an isolated experience. As such, Dr. Lozinski would like to explore ways to extend the PBL experience, perhaps by introducing PBL to the first year Arts and Science calculus course.
Appendix A – Retention question delivered one year after completion of course

Question:
At the train station, free local newspapers have been handed out to 5% of the people present. Assume that 90% of those people are still carrying around those newspapers. Of the people that did not get a free newspaper, 8% of them are carrying copies of the newspaper that they bought for themselves. If you find that a random individual is carrying the local newspaper, what is the probability that it is one of the free papers?

Please provide some additional honest feedback:
Do you remember learning how to do a question like this in ArtSci2R03?

To what extent do you feel that you were able to use concepts learned in ArtSci2R03 to answer the question?

Please provide any additional comments that you feel may be of use in interpreting answers, including the nature of any additional courses you took this last year that you feel have influenced your situation. Thank you for your participation!
## Initial: Grade (G) | Formula | Concept | Reasoning | Retention: Grade (G) | Formula | Concept | Reasoning | Initial Minus Retention Grade
--- | --- | --- | --- | --- | --- | --- | --- | ---
1. A3 | 5.2 | 3 | 3 | 3 | 5.1 | 1 | 3 | 3 | 0.1
2. AA | 5.2 | 2 | 3 | 3 | 1 | 1 | 1 | 1 | 4.2
3. A5 | 4 | 1 | 3 | 3 | 3 | 1 | 1 | 3 | 1
4. B1 | 5 | 3 | 2 | 2 | 5 | 3 | 3 | 3 | 0
5. B2 | 5 | 3 | 1 | 2 | 4.9 | 1 | 1 | 3 | 0.1
6. B9 | 5 | 3 | 2 | 2 | 4 | 1 | 1 | 3 | 4
7. B6 | 5.1 | 3 | 3 | 2 | 4 | 1 | 3 | 3 | 1.1
8. B9 | 5.1 | 3 | 3 | 2 | 4.9 | 1 | 3 | 3 | 0.2
9. C2 | 5.1 | 3 | 3 | 2 | 2 | 1 | 1 | 3 | 3.1
10. C3 | 4.9 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 3.9
11. D5 | 5.1 | 3 | 3 | 3 | 4 | 2 | 3 | 3 | 1.1
12. D6 | 5 | 2 | 3 | 3 | 4 | 2 | 3 | 3 | 1
13. D8 | 5.1 | 3 | 3 | 2 | 5 | 1 | 3 | 3 | 0.1
14. E1 | 2 | 1 | 1 | 2 | 3 | 1 | 3 | 3 | -1
15. E3 | 5.1 | 3 | 3 | 3 | 1 | 1 | 1 | 4.1
16. E4 | 4 | 1 | 2 | 1 | 5 | 1 | 3 | 3 | -1
17. E9 | 5.1 | 3 | 3 | 2 | 5 | 1 | 3 | 3 | 0.1
18. F2 | 4.8 | 1 | 2 | 3 | 0 | 0 | 1 | 1 | 4.8
19. F3 | 5.1 | 3 | 2 | 2 | 5 | 2 | 3 | 2 | 0.1
20. F6 | 4.8 | 3 | 1 | 1 | 0 | 0 | 1 | 1 | 4.8
21. F7 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 3 | 2
22. F8 | 5.1 | 1 | 3 | 3 | 4 | 1 | 3 | 3 | 1.1

### Average

- Grade: 4.7636
- 2.4545
- 2.4545
- 2.3636
- 3.1773
- 1.1818
- 2.1364
- 2.3182
- 1.5864

### Variance

- 0.5253
- 3.3780
- 1.7003

### Standard Deviation

- 0.7248
- 1.8379
- 1.9236

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0-1 No use of formula/demonstration of understanding of concepts/mathematical reasoning

2 Some use of formula/demonstration/understanding of concepts/mathematical reasoning

3 Successfully used formula/demonstrated understanding of concepts/used mathematical reasoning
Appendix C – Sample PBL Exercises

Small Group Exercise 1: Conditional Probability

Suppose the Arts and Science dept and the Engineering dept have been teaming up to pick one random second-year student to be "student of the day" each day since the beginning of the term (5 days a week for 12 weeks). Each day they roll one die to choose which department the student will come from. If they roll a "1", a random Arts and Science student is selected, any other number and it's a random student from engineering. Students could be randomly selected multiple times. The percentage of women in the engineering department is "p".

a) On how many days do you expect the student to be female?

b) If today's student is a female, what is the probability she is from Arts and Science?

c) If this program ran for only x days, and today's student was female, what's the probability that she is from Arts and Science? Does the value of "x" change your answer?

d) Suppose this ran only once, for one day. If today's student is female, what is the probability she is from Arts and Science?

e) If A is the event that the student is chosen from Arts and Science, and B is the event that the student chosen is female, can you write a formula for your answer?

f) Suppose there were 5 departments involved, with A₁ being the event the department chosen was Arts and Science, and A₂ through A₅ being the events that each of the other departments were chosen (B still being the event the student is female). If today's student is female, write a formula for the probability she is in Arts and Science.
Small Group Exercise 5: Sample Means

Assume (based on some data found on the web) that the American male population of age 30 to 35 has an average weight of 175 pounds, and is normally distributed with a standard deviation of 45 pounds. Three people have claimed to have done surveys of randomly sampled males of that age.
Ann sampled 3 people, and found an average of 193 pounds.
Beth sampled 10 people, and found an average of 163 pounds.
Chris sampled 40 people, and found an average of 185 pounds.

You find that one of the surveys was not done correctly. Which of the above three results is most unlikely?

You are to do a survey of the heights of female students at McMaster University. If heights have a standard deviation of 3 inches, how many students would you need to survey in order for the average of your sample to be within 1 inch of the actual population mean with a probability of 90%?
Works Cited


