MATH 3GR3 Assignment #5

Due: Friday, November 24 by 11:59pm

1. Suppose that G is a cyclic group and that N is a subgroup of G. Show that G/N is also a cyclic group.

2. Let G and H be groups and let $M \trianglelefteq G$ and $N \trianglelefteq H$.

   (a) Show that the map $f : G \times H \to G/M \times H/N$ defined by $f((g,h)) = (gM, hN)$ is an onto group homomorphism.

   (b) Show that the kernel of f is $M \times N$.

   (c) Prove that $(G\times H)/(M\times N)$ is isomorphic to $G/M\times H/N$. (Hint: Use the First Isomorphism Theorem.)

3. Determine which of the following maps are group homomorphisms. For those that are, compute their kernels.

   (a) For $n > 1$, $f : \mathbb{Z} \to \mathbb{Z}_n$ is defined by $f(m) = [m]_n$.

   (b) $f : \mathbb{R}^* \to \mathbb{Z}_2$ defined by $f(r) = 0$ if $r > 0$ and $f(r) = 1$ if $r < 0$.

   (c) $f : \mathbb{Q} \to \mathbb{Q}$ defined by $f(q) = |q|$.

4. Let F be the group of all functions $f : \mathbb{Z} \to \mathbb{Z}$ with group operation $+$ defined by $(f + g)(x) = f(x) + g(x)$. For this problem you do not need to show that F is a group under this operation. Use the First Isomorphism Theorem to show that $N = \{f \in F\colon f(3) = 0\}$ is a normal subgroup of F and that F/N is isomorphic to $\mathbb{Z}$.

5. Let $N = \{-1, 1\}$, a subgroup of the group $\mathbb{Q}^*$, and let $\mathbb{Q}^+$ be the subgroup of $\mathbb{Q}^*$ consisting of all positive rational numbers. Use the First Isomorphism Theorem to show that $\mathbb{Q}^*/N$ is isomorphic to $\mathbb{Q}^+$ by constructing a surjective homomorphism from $\mathbb{Q}^*$ to $\mathbb{Q}^+$ that has kernel N.

6. Let G be a group and N a normal subgroup of G. Show that if $aba^{-1}b^{-1} \in N$ for all a, $b \in G$, then the factor group G/N is abelian. Is the converse true?

Supplementary problems from the textbook (not to be handed in)

From Chapter 11, questions 2, 3, 4, 6, 8, 9, 10, 13, 16