

MATH3XO3 (Complex Analysis) Spring 2010

Test 1

February 9, 2010, 11:30AM-12:20PM (50 min.)

Instructor: Kiumars Kaveh

1.

- (a) [5 marks] Find all the solutions of $\sin(z) = i$.
- (b) [5 marks] Find all the solutions of $\bar{z}^3 = 2$. Write them in the form $a + ib$.

2.

- (a) [5 marks] State any of the equivalent definitions of a compact set (in \mathbb{R}^2).
- (b) [5 marks] Let $f : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{R}$ be the function $f(z) = \arg(z)$ where $-\pi \leq \arg(z) < \pi$. Find a connected subset $A \subset \mathbb{C} \setminus \{0\}$ such that $f(A)$ is not connected. (Use the definition of connectedness to show that $f(A)$ is not connected.)

3.

- (a) [6 marks] Show that the function $u(x, y) = \log(\sqrt{x^2 + y^2})$ is harmonic on $A = \mathbb{C} \setminus \{0\}$.
- (b) [4 marks] Find a harmonic conjugate $v(x, y)$ for $u(x, y)$ on the region $\mathbb{C} \setminus \{x + i0 \mid x \leq 0\}$.

4. [10 marks] Find the integral $\int_{\gamma} \operatorname{Re}(z) + z^3$ where the curve γ is the union of three straight line segments joining 1 to i , and i to -1 and finally -1 back to 1 (the image of γ is a triangle). Hint: compute $\int \operatorname{Re}(z)$ and $\int z^3$ separately.

5. [10 marks] Find the region where $\sqrt{\log(z) + 1}$ is analytic (where the square root and $\log(z)$ are principal branches).