MATH3XO3 (Complex Analysis) Spring 2010

Problem Set 2

Due on Thursday March 4 in tutorial.

Instructor: Kiumars Kaveh

1. Compute each of the following integrals (e.g. directly using definition, Fundamental Theorem of Calculus or Cauchy's theorem).

(a) $\int_\gamma (\sin(z) + 1/z)dz$, where γ is the line segment joining 2 to i.

(b) $\int_\gamma \frac{dz}{z^2-1}$, where γ is the circle of radius 1 centered at 1. Hint: use partial fractions.

2. Compute the integral $\int_\gamma \frac{dz}{1+z^2}$, where γ is the rectangle with vertices $(3+3i)$, $(3-3i)$, $(-3+3i)$, $(-3-3i)$ and traversed counter-clockwise. Hint: find the points at which $1/(1+z^2)$ is not analytic and compute the integral of $1/(1+z^2)$ around small circles (say of radius 1) centered at these points. Cite any theorems you use from the text.

3.

(a) Give a parametrization of the ellipse $(x-1)^2 + y^2/4 = 1$.

(b) Give a homotpoy between this ellipse and the point $z_0 = (1, 0)$ (inside some domain G containing the ellipse).

4. Prove using definition (Definition 2.3.9) that the set $G = \{z \mid 1/4 < |z - 1/4| < 3$ is not convex. Use Cauchy's theorem (Theorem 2.3.14) to show that the unit circle centered at the origin is not homotopic to a point in G (and hence G is not simply connected).

5. Suppose $f : \mathbb{C} \to \mathbb{C}$ is an entire function. Moreover, assume that for any $z \in \mathbb{C}$ we have $f(z) = f(z+1) = f(z+i)$. Use Liouville's theorem (Theorem

2.4.8) to prove that f is necessarily a constant function. (Cite any other theorem that you use from the text.)

6. Evaluate $\int_\gamma \frac{\sin(e^z)}{z} dz$, where γ is the unit circle (centered at the origin).