

## MATH3XO3 (Complex Analysis) Spring 2010

### Some practice problems for final

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1. True or False? Justify.

- If  $f$  is an entire function and  $\lim_{z \rightarrow \infty} f(z) = 1$  then  $f$  is the constant function  $f = 1$ .
- If  $f$  is an entire function, for any  $c \in \mathbb{C}$ , the equation  $f(z) = c$  has finite number of solutions.
- Suppose  $f$  is analytic in an open disk  $A$  except at one point  $z_0 \in A$ . Also suppose  $\lim_{z \rightarrow z_0} f(z)$  exists. Then  $f$  has a removable singularity at  $z_0$ .
- Any power series  $f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n$  is an analytic function inside its circle of convergence.
- For any power series  $\sum_{n=0}^{\infty} a_n(z - z_0)^n$  the circle of convergence has radius  $R > 0$ .
- Let  $f$  and  $g$  be two functions which are analytic in some open region  $A$ . Suppose  $A$  contains a simple closed curve  $\gamma$  and  $f(z) = g(z)$  for every  $z \in \gamma$ . Then  $f$  and  $g$  are equal at every point inside  $\gamma$ .
- Any closed curve in  $A = \mathbb{C} \setminus \{0\}$  is homotopic to the unit circle (traversed counter-clockwise).

2. Find the singularities for the following functions:

(a)  $f(z) = \frac{\cos(z)-1}{z^2}$ .

(b)  $f(z) = \frac{z+1}{z^2+9}$ .

(c)  $f(z) = (1 - z^2)e^{1/z}$ .

Determine the type of singularity i.e. removable, pole or essential singularity. For poles determine their order. Find the residue at each singularity.

3. Compute the integrals:

- $\int_{\gamma} \frac{\cos z}{z^2} dz$ , where  $\gamma$  is the unit circle parametrized counterclockwise.
- $\int_{\gamma} \frac{z dz}{z^2 - 2z + 2}$ , where  $\gamma$  is the square with vertices  $\pm 2 \pm 2i$  and parametrized counterclockwise.
- $\int_{\gamma} (1 - z^2)e^{1/z} dz$  where  $\gamma$  is the unit circle parametrized counterclockwise.

4. Find the radius of convergence:

- $\sum_{n=0}^{\infty} \frac{z^{3n}}{27^n}$ .
- $\sum_{n=0}^{\infty} \frac{z^n}{1+2^n}$ .

5. For what complex values of  $z$  is  $\sin(z) = \cos(z)$ ?

6. Find all the values of  $\sqrt[4]{-6 - 6\sqrt{3}i}$ .

7. Compute the Taylor series of  $f(z) = \log(3+z)$  at  $z = 0$ , where  $\log$  denotes the principal branch of the logarithm. What is the radius of convergence of this Taylor series?

8. Find the maximum of  $|\cos(z)|$  for  $|z| \leq 2$ .

9. Use Cauchy-Riemann relations to show that the function  $f(z) = \bar{z}^2$  is not analytic.

10. Write the function  $\sin(z)$  as  $u(x, y) + iv(x, y)$  where  $z = x + iy$ . That is, find the real and imaginary components  $u$  and  $v$  for  $\sin(z)$ .

11. Find the Taylor series of  $e^z + 1/(z - 3)$ , at  $z_0 = 1$ .

12. Find the Laurent series of  $f(z) = \frac{e^{1/z} + 1}{z} + \frac{1}{z-1}$  in  $0 < |z| < 1$ .