

MATH 2R03: LINEAR ALGEBRA II  
\*\*CORRECTED\*\* HOMEWORK 1  
DUE: FRIDAY 18 SEPTEMBER 2009

This homework is a quick review of course material from Math 1B03 in addition to the new material on inner products.

**Required problems (to be handed in):**

1. Let  $V$  be the real vector space  $\mathbb{R}^3$ . Consider the usual dot product in  $\mathbb{R}^3$ .

- (a) Express the dot product  $w \cdot v$  of the vector

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

with the vector

$$v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Your formula should be a linear polynomial in the 3 real variables  $w_1, w_2$ , and  $w_3$ . Prove/justify your answer.

- (b) Find a  $1 \times 3$  real matrix  $A$  so that the matrix multiplication agrees with the projection function, i.e.

$$\frac{(Aw)}{\|v\|^2}v = \text{proj}_v(w).$$

Prove/justify your answer.

2. Let  $v \in \mathbb{R}^3$  be a vector in 3-space. Let  $\text{orth}_v(w)$  be the function from  $\mathbb{R}^3$  to  $\mathbb{R}^3$  which computes the vector component of  $w$  **orthogonal** to  $v$ , so

$$\text{orth}_v(w) = w - \text{proj}_v(w).$$

Remember that  $\mathbb{R}^3$  is a vector space, and if  $w_1, w_2$  are two vectors in  $\mathbb{R}^3$ , then  $w_1 + w_2$  denotes their vector sum. Also, if  $c$  is a real number and  $w$  is a vector in  $\mathbb{R}^3$ , then  $cw$  denotes the scalar multiplication of  $w$  by the constant  $c$ .

- (a) Let  $w_1, w_2 \in \mathbb{R}^3$ . Is it true that

$$\text{orth}_v(w_1 + w_2) = \text{orth}_v(w_1) + \text{orth}_v(w_2)?$$

Prove/justify your answer.

- (b) Let  $w \in \mathbb{R}^3$  and  $c \in \mathbb{R}$ . Is it true that

$$\text{orth}_v(cw) = c\text{orth}_v(w)?$$

Prove/justify your answer.

- (c) Does your answer to the first two parts of this question automatically imply that

$$\text{orth}_v(c_1w_1 + c_2w_2) = c_1\text{orth}_v(w_1) + c_2\text{orth}_v(w_2)?$$

Prove/justify your answer.

3. Anton-Rorres, Chapter 6, Section 1: 22.

**Recommended problems (not to be handed in):**

1. Anton-Rorres, Chapter 6, Section 1: 17

2. Anton-Rorres, Chapter 6, Section 1: 20

3. Anton-Rorres, Chapter 6, Section 1, Discussion Discovery: 34.