Tutorial #9

November 16, 2015 5:23 PM

1. $$0 = \begin{vmatrix} -1-\lambda & 2 \\ 0 & -3-\lambda \end{vmatrix}$$

$$= \underbrace{(-1-\lambda)(-3-\lambda)} \Rightarrow \lambda = -1$$

$$\underline{\lambda = -3.}$$

$$0 = 3 + 4\lambda + \lambda^2$$

$$0 = 3I + 4A + A^2$$

$$\underline{A^2 = -4A - 3I}$$

$$\left[\begin{array}{l} A^K = c_0 I + c_1 A \\ \lambda^K = c_0 + c_1 \lambda \end{array}\right]$$

$$(-3)^K = c_0 - 3c_1$$

$$(-1)^k = c_0 - c_1 \quad *$$

$$(-3)^k - (-1)^k = -2c_1$$

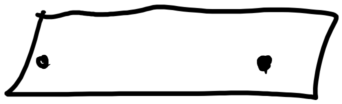
$$c_1 = -\frac{(-3)^k + (-1)^k}{2}.$$

$$c_0 = (-1)^k + c_1$$

$$= -\frac{1}{2}(-3)^k + \frac{3}{2}(-1)^k.$$

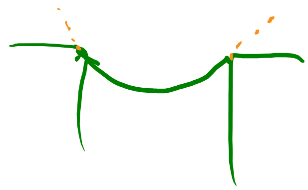
$$A^k = c_0 I + c_1 A$$

$$= \begin{bmatrix} (-1)^k & -(-3)^k + (-1)^k \\ 0 & (-3)^k \end{bmatrix}.$$

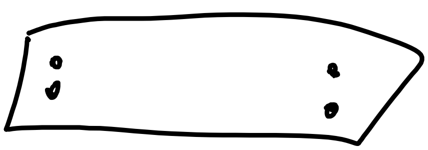
2. (a) 

hinged on both sides

$$\begin{cases} y(0)=0 \\ y''(0)=0 \end{cases}$$



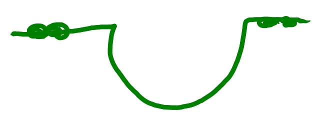
$$\begin{cases} y(L)=0 \\ y''(L)=0. \end{cases}$$

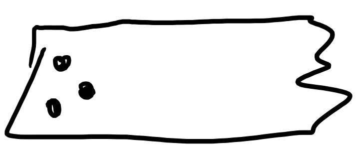
(b) 

embedded on both sides

$y(0)=0$ $\quad$ $y(L)=0$

$y'(0)=0$ $\quad$ $y'(L)=0$



(c) 

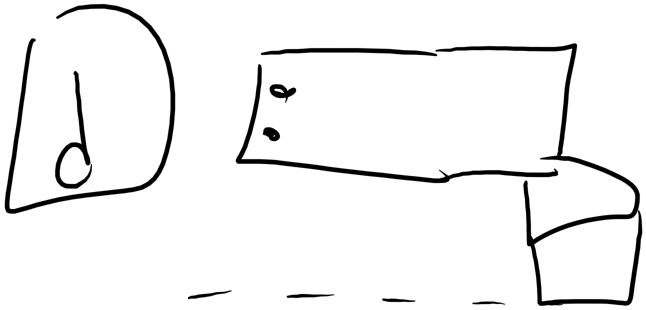
Left embedded

Right Free

$y(0)=0$ $\quad$ $y''(L)$

Right free

$$\begin{cases} y(0)=0 \\ y'(0)=0 \end{cases} \qquad \begin{aligned} y''(L)&=0 \\ y'''(L)&=0. \end{aligned}$$



Left embedded

Right simply supported

$$\boxed{\begin{cases} y(0)=0 \\ y'(0)=0 \end{cases} \qquad \begin{cases} y(L)=0 \\ y''(L)=0. \end{cases}}$$

3. $$y^{(4)} = \frac{w_0}{EI}$$

$$m^4 = 0$$

$$y_c = c_1 + c_2 x + c_3 x^2 + c_4 x^3.$$

$$y_p = A x^4$$

$$y_p^{(4)} = 24A.$$

$$24A = \frac{w_0}{EI}$$

$$A = \frac{w_0}{24EI}.$$

$$\therefore y = y_c + y_p = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \frac{w_0}{24EI} x^4.$$

Plug boundary condition in & get: Find c_1, c_2, c_3, c_4

$$y = \frac{w_0}{48EI}\left[3L^2x^2 - 5Lx^3 + 2x^4\right].$$

Test 2013: [#4]

(a) $mx'' = -kx - \beta x' + F(t)$

$m = 1$ $\quad F(t) = 8\sin(2t)$

$k = 12$

$\beta = 6$

$$\boxed{x'' + 12x + 6x' = 8\sin(2t)}$$

b) $\beta = 0$

$$x'' + 12 = A\cos(\omega t).$$

$$m^2 + 12 = 0$$

$$m = \pm\sqrt{12}\, i = \pm 2\sqrt{3}\, i.$$

$$x_c = c_1 \cos(2\sqrt{3}t) + c_2 \sin(2\sqrt{3}t).$$

$$x_p = \{B\cos(2\sqrt{3}t) + C\sin(2\sqrt{3}t)\}t$$ ✓

if $\boxed{\omega = 2\sqrt{3}}$

If $\omega \neq 2\sqrt{3}$, then

$$x_p = [B\cos(\omega t) + C\sin(\omega t)]$$

Want: Either $B \neq 0$ or $C \neq 0$ or both.

$$x_p' = B\cos(2\sqrt{3}t) + C\sin(2\sqrt{3}t)$$

$$+ t\,[-2\sqrt{3}B\sin(2\sqrt{3}t) \pm$$

$$+ t \left[-2\sqrt{3} B \sin(2\sqrt{3}\, t) + 2\sqrt{3}\, C \cos(2\sqrt{3}\, t) \right]$$

$$x_p'' = -2 \cdot 2\sqrt{3} B \sin(2\sqrt{3}\, t) + 2 \cdot 2\sqrt{3}\, C \cos(2\sqrt{3}\, t) + t \left[-B (2\sqrt{3})^2 \cos(2\sqrt{3}\, t) - C (2\sqrt{3})^2 \sin(2\sqrt{3}\, t) \right]$$

$$x_p'' + 12 x_p = A \cos(2\sqrt{3}\, t)$$

$$\Rightarrow \underline{B = 0}, \quad C = \frac{A}{4\sqrt{3}}$$

$$\Rightarrow A \neq 0.$$