Math 2Z03 - Tutorial #9

Nov. 16th, 17th, 18th, 2015

Tutorial Info:

- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)

Tutorial #8:

- 8.9 Powers of Matrices (Cayley-Hamilton Theorem)
- 3.9 Linear Models: Boundary-Value Problems (Deflection of a Beam)
- 10.2 Homogeneous Linear Systems

8.9 Powers of Matrices (Cayley-Hamilton Theorem)

- **Last Tutorial:** We found A^k of a matrix using diagonalization.
- **1.** Find A^k using the Cayley-Hamilton Theorem, where

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix}.$$

- **Cayley-Hamilton Theorem:** An $n \times n$ matrix A satisfies its own characteristic equation.

3.9 Linear Models: Boundary-Value Problems (Deflection of a Beam)

- **Recall:** The **deflection of a beam** can be modelled by the DE

$$EIy^{(4)} = w(x),$$

where $w(x)$ is the load per unit length, E and I are constants, and $y(x)$ is the deflection.

- **Recall:** A beam can have various **boundary conditions**:
  - **Free:** $y'' = 0$, $y''' = 0$
  - **Embedded:** $y = 0$, $y' = 0$
  - **Simply Supported of Hinged:** $y = 0$, $y'' = 0$

3.9 Linear Models: Boundary-Value Problems (Deflection of a Beam)

- **2.** Suppose a shopkeeper wants to put up a rectangular sign of length L for his store, and that the deflection of the sign can be modelled by the fourth-order DE $EIy^{(4)} = w(x)$. Identify the appropriate boundary conditions for the following cases:
  - a) He uses one nail on each side.
  - b) He uses two nails on each side.
  - c) He uses three nails on the left side and no nails on the right side.
  - d) He uses two nails on the left side and a stack of crates on the right side.
- **3.** Find the deflection, $y(x)$, in d) if $w(x) = w_0$ a constant, and $0 < x < L$.
- **Recall:** A beam can have various **boundary conditions**:
  - **Free:** $y'' = 0, y''' = 0$
  - **Embedded:** $y = 0, y' = 0$
  - **Simply Supported of Hinged:** $y = 0, y'' = 0$

10.2 Homogeneous Linear Systems

- **4. a)** Solve the homogeneous system of linear DE's:

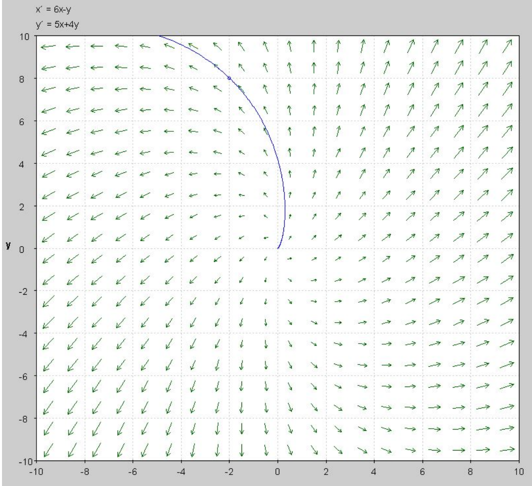
$$X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X, X(0) = \left[\begin{smallmatrix} -2 \\ 8 \end{smallmatrix}\right].$$

- **Recall:** Consider the homogeneous linear DE $X' = AX$. If $\lambda = \alpha + \beta i$ is an eigenvalue of the coefficient matrix A, with corresponding eigenvector $v = B_1 + B_2 i$, then two linearly independent solutions of this system on $(-\infty, \infty)$ are:

$$X_1 = e^{\alpha t}\left[B_1 \cos(\beta t) - B_2 \sin(\beta t)\right]$$
$$X_2 = e^{\alpha t}\left[B_1 \sin(\beta t) + B_2 \cos(\beta t)\right].$$

- **b)** Sketch the solution curve corresponding to this IVP.

10.2 Homogeneous Linear Systems



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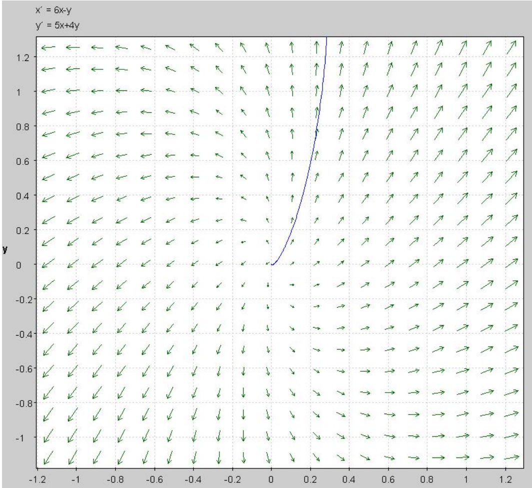

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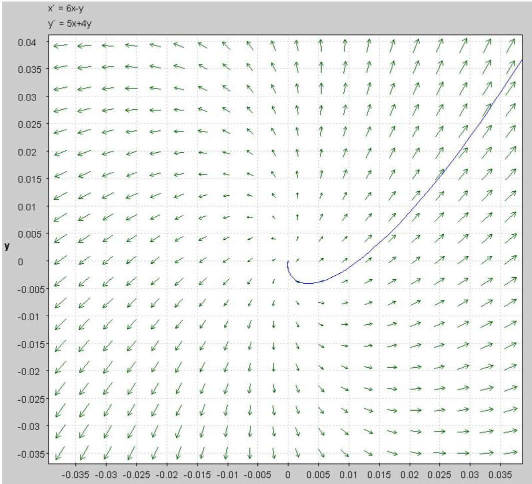

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