Math 2Z03 - Tutorial #9

Nov. 16th, 17th, 18th, 2015

Tutorial Info:

- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)

Tutorial #8:

- 8.9 Powers of Matrices (Cayley-Hamilton Theorem)

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- **Cayley-Hamilton Theorem:** An $n \times n$ matrix A satisfies its own characteristic equation.

3.9 Linear Models: Boundary-Value Problems (Deflection of a Beam)

- **Recall:** The **deflection of a beam** can be modelled by the DE

$$EIy^{(4)} = w(x),$$

where $w(x)$ is the load per unit length, E and I are constants, and $y(x)$ is the deflection.

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- **Recall:** A beam can have various **boundary conditions**:
  - **Free:** $y'' = 0$, $y''' = 0$
  - **Embedded:** $y = 0$, $y' = 0$
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- **2.** Suppose a shopkeeper wants to put up a rectangular sign of length L for his store, and that the deflection of the sign can be modelled by the fourth-order DE $EIy^{(4)} = w(x)$. Identify the appropriate boundary conditions for the following cases:

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- **3.** Find the deflection, $y(x)$, in d) if $w(x) = w_0$ a constant, and $0 < x < L$.
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10.2 Homogeneous Linear Systems

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$$X_1 = e^{\alpha t}\left[B_1 \cos(\beta t) - B_2 \sin(\beta t)\right]$$
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- **b)** Sketch the solution curve corresponding to this IVP.

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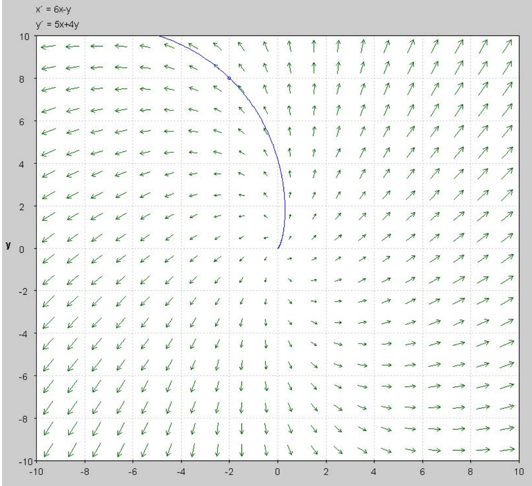

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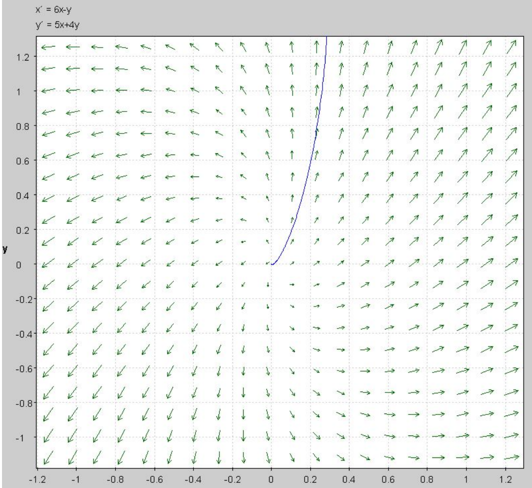

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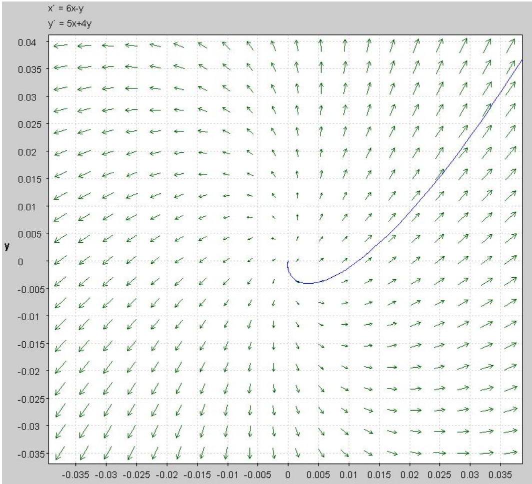

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