Math 2Z03 - Tutorial # 8

Nov. 9th, 10th, 11th, 2015

Tutorial Info:

- **Review Session:** Monday Nov. 16th, 7-9pm, JHE264
- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)

Tutorial #8:

- 3.9 Linear Models: Boundary-Value Problems (Eigenfunctions)
- 8.8 Eigenvalues and Eigenvectors
- 8.9 Powers of Matrices (Cayley-Hamilton Theorem)
- 8.10 Symmetric and Orthogonal Matrices
- 8.12 Diagonalization

3.9 Linear Models: Boundary-Value Problems (Eigenfunctions)

- **1.** Find the eigenvalues and eigenfunctions for the BVP $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$.
- **Recall:** If the homogeneous BVP involves a parameter λ, then the values of λ for which it has at least one nontrivial solution (i.e. not the zero solution) are called **eigenvalues** and the corresponding functions are called **eigenfunctions**.

8.8 Eigenvalues and Eigenvectors

- **2.** Consider $A = \begin{pmatrix} 8 & 9 \\ -6 & 7 \end{pmatrix}$.
- **a)** What are the eigenvalues of A?
- **Recall:** If A is square, then a nonzero vector x is called an **eigenvector** of A if $Ax = \lambda x$ for some scalar λ. The scalar λ is called an **eigenvalue** of A and x is its corresponding eigenvector.
- **b)** Find all eigenvectors of A.
- **c)** Is A invertible?
- **Recall:** A is invertible if and only if $\lambda = 0$ is NOT an eigenvalue of A.

8.8 Eigenvalues and Eigenvectors

- **3.** Consider $A = \begin{pmatrix} 5 & -3 \\ a & b \end{pmatrix}$. Suppose $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector of A. What must the eigenvalue corresponding to x be?

8.9 Powers of Matrices (Cayley-Hamilton Theorem)

- **4.** Consider $B = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

  Find B^{-1} using the Cayley-Hamilton Theorem.

- **Cayley-Hamilton Theorem:** An $n \times n$ matrix A satisfies its own characteristic equation.

8.10 Symmetric and Orthogonal Matrices

- **5.** Consider $C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.
- **a)** Is C orthogonal?
- **Recall:** An $n \times n$ nonsingular matrix A is orthogonal if $A^{-1} = A^T$.
- **b)** Is C symmetric?
- **Recall:** An $n \times n$ matrix A is symmetric if $A = A^T$.
- **c)** Does C have real eigenvalues?
- **Theorem 8.10.1:** If A is a symmetric matrix with real entries, then the eigenvalues of A are real.

8.12 Diagonalization

- **6.** Consider $A = \begin{pmatrix} -2 & -27 & 9 \\ 0 & -2 & 0 \\ 0 & -18 & 4 \end{pmatrix}$. Find A^k.