

Math 2Z03 - Tutorial # 8



Nov. 9th, 10th, 11th, 2015

Tutorial Info:

- **Review Session:** Monday Nov. 16th, 7-9pm, JHE264
- **Tutorial Website:** <http://ms.mcmaster.ca/~dedieula/2Z03.html>
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)



Tutorial #8:

- 3.9 Linear Models: Boundary-Value Problems (Eigenfunctions)



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- 8.8 Eigenvalues and Eigenvectors



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- 8.9 Powers of Matrices (Cayley-Hamilton Theorem)
- 8.10 Symmetric and Orthogonal Matrices



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- 3.9 Linear Models: Boundary-Value Problems (Eigenfunctions)
- 8.8 Eigenvalues and Eigenvectors
- 8.9 Powers of Matrices (Cayley-Hamilton Theorem)
- 8.10 Symmetric and Orthogonal Matrices
- 8.12 Diagonalization



3.9 Linear Models: Boundary-Value Problems (Eigenfunctions)

- 1. Find the eigenvalues and eigenfunctions for the BVP $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$.



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- 1. Find the eigenvalues and eigenfunctions for the BVP $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$.
- **Recall:** If the homogeneous BVP involves a parameter λ , then the values of λ for which it has at least one nontrivial solution (i.e. not the zero solution) are called **eigenvalues** and the corresponding functions are called **eigenfunctions**.



8.8 Eigenvalues and Eigenvectors

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- a) What are the eigenvalues of A ?



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- b) Find all eigenvectors of A .



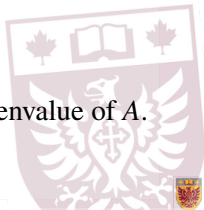
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- b) Find all eigenvectors of A .
- c) Is A invertible?



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- c) Is A invertible?
- **Recall:** A is invertible if and only if $\lambda = 0$ is NOT an eigenvalue of A .



8.8 Eigenvalues and Eigenvectors

- 3. Consider $A = \begin{pmatrix} 5 & -3 \\ a & b \end{pmatrix}$. Suppose $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector of A .
What must the eigenvalue corresponding to x be?



8.9 Powers of Matrices (Cayley-Hamilton Theorem)

- 4. Consider $B = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

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- **Cayley-Hamilton Theorem:** An $n \times n$ matrix A satisfies its own characteristic equation.



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■ c) Does C have real eigenvalues?

■ **Theorem 8.10.1:** If A is a symmetric matrix with real entries, then the eigenvalues of A are real.



8.12 Diagonalization

- 6. Consider $A = \begin{pmatrix} -2 & -27 & 9 \\ 0 & -2 & 0 \\ 0 & -18 & 4 \end{pmatrix}$. Find A^k .

