Math 2Z03 - Tutorial # 5

Oct. 19th, 20th, 21st, 2015

Tutorial Info:

- **Review Session:** Tuesday, Oct. 20th, 4:30pm-6:30pm (MDCL 1105)
- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)

Tutorial #5:

- 3.1 Theory of Linear Equations
  - Existence Uniqueness
- 3.8 Linear Models: IVP's
  - Spring/Mass Systems: Free Undamped Motion

3.1 Theory of Linear Equations

- **1.** Find the largest interval for which Theorem 3.1.1 guarantees that the given IVP has a unique solution.
- **Theorem 3.1.1 (Linear Existence/Uniqueness):**Consider the n-th order linear IVP

$$a_n(x)y^{(n)} + \cdots + a_1(x)y' + a_0(x)y = g(x)$$
$$y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}.$$

  Suppose $a_n(x) \neq 0$ on an interval I and that $a_n(x), \ldots, a_0(x), g(x)$ are continuous on I. If x_0 lies on this interval I, then there *exists* a *unique* solution to this IVP on the entire interval I.

- **a)** $(x^2-1)y'' + 3xy' + \cos xy = e^x$, $y(0) = 4$, $y'(0) = 5$.
- **b)** $\ln xy' + y = \cot x$, $y(2) = 3$.

3.8 Spring/Mass Systems: Free Undamped Motion

- **2.** A mass weighing 64lb stretches a spring 0.32ft. The mass is initially released from a point 8 inches above the equilibrium position with downward velocity of 5 ft/s.

  a) Find the equation of motion.

  b) What is the amplitude and period of motion?

  c) How many complete cycles will the mass have completed by the end of 3π second?

  d) At what time does the mass pass through the equilibrium position heading downward for the second time?

  e) At what time does the mass attain its extreme displacement on either side of the equilibrium position?

  f) What is the position of the mass at $t = 3s$?

3.8 Spring/Mass Systems: Free Undamped Motion

- **2.** A mass weighing 64lb stretches a spring 0.32ft. The mass is initially released from a point 8 inches above the equilibrium position with downward velocity of 5 ft/s.

  g) What is the instantaneous velocity at $t = 3s$?

  h) What is the acceleration at $t = 3s$?

  i) What is the instantaneous velocity at the times when the mass passes through the equilibrium position?

  j) At what times is the mass 5 inches below the equilibrium position?

  k) At what times is the mass 5 inches below the equilibrium position heading in the upward direction?

3.8 Spring/Mass Systems: Free Undamped Motion

- **3.** A mass weighing 32lb is suspended from a spring whose spring constant is 9lb/ft. The mass is initially released from a point 1ft above the equilibrium position with an upward velocity of $\sqrt{3}$ft/s. Find the times for which the mass is heading downward at a velocity of 3ft/s.