Math 2Z03 - Tutorial # 5

Oct. 19th, 20th, 21st, 2015

Tutorial Info:

- **Review Session:** Tuesday, Oct. 20th, 4:30pm-6:30pm (MDCL 1105)
- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)

Tutorial #5:

- 3.1 Theory of Linear Equations

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  - Spring/Mass Systems: Free Undamped Motion

3.1 Theory of Linear Equations

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- **Theorem 3.1.1 (Linear Existence/Uniqueness):** Consider the n-th order linear IVP

$$a_n(x)y^{(n)} + \cdots + a_1(x)y' + a_0(x)y = g(x)$$
$$y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}.$$

Suppose $a_n(x) \neq 0$ on an interval I and that $a_n(x), \ldots, a_0(x), g(x)$ are continuous on I. If x_0 lies on this interval I, then there *exists* a *unique* solution to this IVP on the entire interval I.

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- **a)** $(x^2 - 1)y'' + 3xy' + \cos xy = e^x$, $y(0) = 4$, $y'(0) = 5$.
- **b)** $\ln xy' + y = \cot x$, $y(2) = 3$.

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  f) What is the position of the mass at $t = 3s$?

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  k) At what times is the mass 5 inches below the equilibrium position heading in the upward direction?

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- **3.** A mass weighing 32lb is suspended from a spring whose spring constant is 9lb/ft. The mass is initially released from a point 1ft above the equilibrium position with an upward velocity of $\sqrt{3}$ft/s. Find the times for which the mass is heading downward at a velocity of 3ft/s.