

Math 2Z03 - Tutorial # 4



Oct. 5th, 6th, 7th, 2015

Tutorial Info:

- **Tutorial Website:** <http://ms.mcmaster.ca/~dedieula/2Z03.html>
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)



Tutorial #4:

- 3.1 Theory of Linear Equations



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 - Linearly Independence



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 - Finding Rational Roots



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- 3.1 Theory of Linear Equations
 - Linearly Independence
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 - Fundamental Set of Solutions
 - General Solution

- 3.3 Homogeneous Linear Equations with Constant Coefficients
 - Finding Rational Roots
 - Finding Complex Roots



3.1 Theory of Linear Equations

- 1. Is the set of functions $\{1 + x, x, x^2\}$ linearly independent on $(-\infty, \infty)$?



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- **Recall:** A set of functions $f_1(x), \dots, f_n(x)$ are **linearly independent** on an interval I if $c_1f_1(x) + \dots + c_nf_n(x) = 0$ for all x in $I \iff c_1 = \dots = c_n = 0$.



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- **Criterion for Linear Independent Solutions:** Let y_1, \dots, y_n be solutions of a homogeneous linear n -th order DE on an interval I . Then this set of solutions is linearly independent on I if and only if the Wronskian $W(y_1, \dots, y_n) \neq 0$ for every x in I .



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- 2. Suppose f_1, f_2 , and f_3 are solutions to a second-order linear homogeneous differential equation. Is $\{f_1, f_2, f_3\}$ a fundamental set of solutions?



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- **Recall:** A basis for the space of solutions of an n -th order homogeneous linear equation $a_n y^{(n)} + \cdots + a_1 y' + a_0 = 0$ is called a **fundamental set of solutions**.

We know the dimension of the solution space is n , so to find a basis, it suffices to find n linearly independent solutions.



3.1 Theory of Linear Equations

- **3.** The functions e^t and te^t satisfy the differential equation $y'' - 2y' + y = 0$. Is $y = c_1e^t + c_2te^t$ a general solution of this differential equation?



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- **3.** The functions e^t and te^t satisfy the differential equation $y'' - 2y' + y = 0$. Is $y = c_1e^t + c_2te^t$ a general solution of this differential equation?
- **Recall:** If $\{y_1, \dots, y_n\}$ is a fundamental set of solutions on I for an n -th order linear DE, then the **general solution** on I is $y = c_1y_1 + \dots + c_ny_n$, where the c_i are arbitrary constants.



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- **Recall:** To solve, first we plug $y = e^{mx}$ into the equation and find the roots of the corresponding auxiliary equation.



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- **Recall:** To solve, first we plug $y = e^{mx}$ into the equation and find the roots of the corresponding auxiliary equation.
- *We can go about finding the roots of this auxiliary equation in a variety of ways.*



3.3 Homogenous Linear Equations with Constant Coefficients:

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- By inspection, we can see that $m = -2$ is a root. A more systematic approach is given by the following:
- **Rational Roots Test (pg. 122):** If $m_1 = \frac{p}{q}$ is a rational root (expressed in lowest terms) of an auxiliary equation with integer coefficients

$$a_n m^n + \dots + a_1 m + a_0,$$

then p is a factor of a_0 and q is a factor of a_n .



3.3 Homogenous Linear Equations with Constant Coefficients:

- **Method 2: Finding roots over \mathbb{C} :** Given auxiliary equation

$$a_n m^n + \dots + a_1 m + a_0,$$

we know there will be n roots over the complex numbers (with some multiplicity). By working in polar coordinates, we can find these roots $re^{i\theta}$ (think back to Math 1ZC3/1B03).



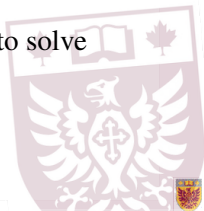
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- **Exercise** Find the five roots of $m^5 + 32 = 0$, and use this to solve $y^{(5)} + 32y = 0$.



3.3 Homogenous Linear Equations with Constant Coefficients:

■ Recall:

- If there are j distinct roots m_1, \dots, m_j then the general solution contains a linear combination of $e^{m_1x}, \dots, e^{m_jx}$.
- If m_1 is a root of multiplicity q , then the general solution contains a linear combination of $e^{m_1x}, xe^{m_1x}, x^2e^{m_1x}, \dots, x^{q-1}e^{m_1x}$.
- Given complex roots $m_1 = \alpha + \beta$ and $m_2 = \alpha - \beta$, using Euler's formula, it's always possible to write $c_1e^{m_1x} + c_2e^{m_2x}$ as $e^{\alpha x} [k_1 \cos(\beta x) + k_2 \sin(\beta x)]$, for constants c_1, c_2, k_1, k_2 .



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- 5. Solve the IVP $y'' + 2y' = 0$, $y(0) = 1$, $y'(0) = 1$.



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- 5. Solve the IVP $y'' + 2y' = 0$, $y(0) = 1$, $y'(0) = 1$.
- 6. Find a general solution for $6y^{(4)} - y''' + 4y'' - y' - 2y = 0$.

