Math 2Z03 - Tutorial # 1

Sept. 14th, 15th, 16th, 2015

Tutorial Info:

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- **Tutorial Website:** http://ms.mcmaster.ca/∼dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (starting Sept. 21st, in the Math Help Centre)

Tutorial #1:

- 1.1 Definitions and Terminology
- 1.2 Initial Value Problems (IVP's)
- 2.1 Solution Curves Without a Solution
  - Direction Fields
  - Autonomous DE's *(NEXT TIME)*

1.1 Definitions and Terminology

- ODE vs. PDE
- Order
- Linear

- **Recall:** An n-th order DE $F(x,y,y',\ldots,y^{(n)})=0$ is **linear** if F is linear in the variables $y,y',\ldots,y^{(n)}$. **i.e.** It can be written in the form

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}}+\ldots+a_1(x)\frac{dy}{dx}+a_0(x)y-g(x)=0.$$

Order and Linearity

- 1. State the order of the following differential equation. Is it linear?

a) $\frac{d^2y}{dx^2} = \sqrt{1+(\frac{dy}{dx})^3}$

b) $(sint)y''' - (cost)y' = 10.$

1.1 Definitions and Terminology

- ODE vs. PDE
- Order
- Linear
- Explicit vs. Implicit Solution

- **Recall:** An **(explicit) solution**, y, to a *first* order DE on an interval $\mathbf{I}$ must be $\mathbf{C^1}$ on I (*continuously differentiable*...i.e. y is defined and y' is continuous) on I AND when substituted into the DE, **reduces the equation to the identity**.

Explicit Solutions

- 2. Is the piece-wise defined function a solution to the given DE on the interval provided? Explain

a)

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases},$$

$xy' - 2y = 0, I = (-\infty, \infty).$

b)

$$y = \begin{cases} \sqrt{25 - x^2}, & -5 < x < 0 \\ -\sqrt{25 - x^2}, & 0 \leq x < 5 \end{cases},$$

$y' = \frac{-x}{y}, I = (-5, 5).$

Explicit Solutions

- 3. Given that $y = sinx$ is an explicit solution of $y' = \sqrt{1-y^2}$, find an interval of definition.

1.2 IVP's

- Existence and Uniqueness
- **Existence of a Unique Solution (1st-Order IVP's):** Let $R = [a,b] \times [c,d]$ contain the point (x_0, y_0) in its interior. If $f(x,y)$ and $\frac{df}{dy}$ are continuous on R, then there exists some interval I_0 containing x_0 contained in $[a,b]$ and a unique function $y(x_0)$ defined on I_0 such that $y(x)$ is a unique solution to the IVP $y' = f(x,y), y(x_0) = y_0$.

Existence/ Uniqueness

- 4. Determine whether the Existence/ Uniqueness Theorem guarantees that the DE $y' = \sqrt{y^2 - 9}$ possesses a unique solution through the given point.

  a) $(1,4)$

  b) $(2,-3)$

Existence/ Uniqueness

- 5. Suppose you are given a first-order differential equation $y' = f(x,y)$, in which $f(x,y)$ and $\frac{df}{dy}$ are continuous in some rectangular region R. Could two different solution curves in its 1-parameter family of solutions intersect at a point in R? Why or why not?

1.3 Solution Curves Without a Solution

- Direction Fields

1.3 Solution Curves Without a Solution

- 6. Consider the first-order differential equations
  a) $y' = (2-y)(3-y)$, **b)** $y' = (y-2)(3-y)$, **c)** $y' = (2-x)(3+x)$, **d)** $y' = (2-y)(3+x)$. Assign the direction fields below to the appropriate differential equation.