Math 2C03: Quiz #6 Information

QUIZ: WEDNESDAY, JULY 29TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Potential Quiz Questions:

Your quiz on Wednesday will consist of one or two of the questions listed below.

1. (a) What does it mean for a function to be analytic at a point x_0?
   (b) Is $x = 0$ an ordinary or singular point of $xy'' + (\sin x)y = 0$? $\left(\textbf{Hint: } \text{Use the Maclaurin series } \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}\right)$.

2. Is $x = 0$ an ordinary point of $y'' + 5xy' + \sqrt{x}y = 0$? Explain.

3. Does the differential equation $(1 + x + x^2)y'' - 3y = 0$ have a power series solution about the point $x = 1$? If so, what is the minimum radius of convergence of this power series. Explain.

   (**Hint:** You don't have to solve this DE to answer this question.)

4. Show that the differential equation $(1 + x^2)y'' - y' + y = 0$ has power series solution $y = \sum_{n=0}^{\infty} c_n x^n$, where the recursive formula for the coefficients c_n is $c_{n+2} = \frac{(n+1)c_{n+1} - (n^2 - n + 1)c_n}{(n+2)(n+1)}$, $n \geq 2$.

5. The differential equation $(1 + x^2)y'' - y' + y = 0$ has power series solution $y = \sum_{n=0}^{\infty} c_n x^n$, where the recursive formula for the coefficients c_n is $c_{n+2} = \frac{(n+1)c_{n+1} - (n^2 - n + 1)c_n}{(n+2)(n+1)}$, $n \geq 2$, $2c_2 - c_1 + c_0 = 0$, $6c_3 - 2c_2 + c_1 = 0$. Using this, write the first 4 terms for a general solution to this DE. How do you know this is a general solution?