Math 2C03: Quiz #5 Information

QUIZ: MONDAY, JULY 27TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Potential Quiz Questions:

Your quiz on Monday will consist of one or two of the questions listed below.

1. For this question, it suffices to draw an example of a function that satisfies the conditions below. (i.e. You don't have to explicitly write down an equation that defines the function.)
   (a) Give an example of a function that is piecewise continuous on $[0, \infty)$. Explain.
   (b) Give an example of a function that is NOT piecewise continuous on $[0, \infty)$. Explain
2. Compute $\mathscr{L}\{te^{-3t}\cos(3t)\}$ using the Laplace transform table provided.
3. Solve
$$t - 2f(t) = \int_0^t (e^{\tau} - e^{-\tau}) f(t-\tau)\, d\tau.$$
4. Solve the following system of linear differential equations:
$$x' = 2x + y$$
$$y' = 3x + 4y$$
$$x(0) = 1, y(0) = 0$$
5. Verify that the power series
$$y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$$
is a solution to the differential equation $xy'' + y' + xy = 0$.
Hint: You'll want to make a substitution $k = n$ and $k = n + 1$.

Table of Laplace Transforms

Here $\mathscr{L}\{f(t)\} = F(s)$.

Section 7.1:

- $\mathscr{L}\{1\} = \dfrac{1}{s}$
- $\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}}, n = 1, 2, \ldots$
- $\mathscr{L}\{e^{at}\} = \dfrac{1}{s-a}$
- $\mathscr{L}\{\sin(kt)\} = \dfrac{k}{s^2+k^2}$
- $\mathscr{L}\{\cos(kt)\} = \dfrac{s}{s^2+k^2}$
- $\mathscr{L}\{\sinh(kt)\} = \dfrac{k}{s^2-k^2}$
- $\mathscr{L}\{\cosh(kt)\} = \dfrac{s}{s^2-k^2}$

Section 7.2:

- $\mathscr{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$

Section 7.3:

- $\mathscr{L}\{e^{at}f(t)\} = \mathscr{L}\{f(t)\} \mid_{s \to s-a} = F(s-a)$, where $a \in \mathbb{R}$
- $\mathscr{L}^{-1}\{F(s-a)\} = \mathscr{L}^{-1}\{F(s) \mid_{s \to s-a}\} = e^{at}f(t)$
- $\mathscr{L}\{f(t-a)\mathscr{U}(t-a)\} = e^{-as}F(s)$, where $a > 0$
- $\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathscr{U}(t-a)$, where $a > 0$
- $\mathscr{L}\{g(t)\mathscr{U}(t-a)\} = e^{-as}\mathscr{L}\{g(t+a)\}$, where $a > 0$
- $\mathscr{L}\{\mathscr{U}(t-a)\} = \dfrac{e^{-as}}{s}$, where $a > 0$

Section 7.4:

- $\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}F(s), n = 1, 2, \ldots$
- $\mathscr{L}\{f * g\} = \mathscr{L}\{f(t)\}\mathscr{L}\{g(t)\} = F(s)G(s)$