# Math 2C03: Quiz #4 Information

QUIZ: MONDAY, JULY 20TH, 7PM (FIRST 10 MINUTES OF CLASS)
*McMaster University*

## Potential Quiz Questions:

Your quiz on Monday will consist of one or two of the questions listed below.

1. (a) State the definition of the Laplace transform.
   (b) Compute the Laplace transform, $\mathscr{L}\left\{e^{2t}\right\}$ using the definition of the Laplace transform (i.e. you'll have to compute an integral here).
2. Using the Laplace transform table provided, compute the following:
   (a) $\mathscr{L}\{\cos(5t)+\sin(2t)\}$.
   (b) $\mathscr{L}^{-1}\left\{\frac{1}{5s-2}\right\}$.
3. Using the Laplace transform table provided, compute the following:
   (a) $\mathscr{L}\{t^2 - e^{-9t} + 5\}$.
   (b) $\mathscr{L}^{-1}\left\{\frac{1}{4s^2+1}\right\}$.
4. Using the Laplace transform table provided, compute $\mathscr{L}^{-1}\left\{\frac{s+8}{(s-1)(s+4)}\right\}$. (Hint: You'll need to use partial fractions.)
5. Solve The IVP $y'' - y' - 6y = 0$, $y(0) = 2$, $y'(0) = -1$.
   (Hint: $\frac{2s-3}{(s-3)(s+2)} = \frac{\frac{3}{5}}{s-3} + \frac{\frac{7}{5}}{s+2}$. This can be computed using partial fractions, but you're not required to do so here.)
6. (a) Write the following function in terms of unit step functions:

$$f(t) = \begin{cases} 1 & : 0 \leq t < 4 \\ 0 & : 4 \leq t < 5 \\ 1 & : t \geq 5 \end{cases}$$

   (b) Using the Laplace Transform table provided, find $\mathscr{L}\{f(t)\}$.

## Table of Laplace Transforms:

Here $\mathscr{L}\{f(t)\} = F(s)$.

**Section 7.1:**

- $\mathscr{L}\{1\} = \frac{1}{s}$
- $\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}$, $n = 1, 2, \ldots$
- $\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$
- $\mathscr{L}\{\sin(kt)\} = \frac{k}{s^2+k^2}$
- $\mathscr{L}\{\cos(kt)\} = \frac{s}{s^2+k^2}$
- $\mathscr{L}\{\sinh(kt) = \frac{k}{s^2-k^2}$
- $\mathscr{L}\{\cosh(kt)\} = \frac{s}{s^2-k^2}$

**Section 7.2:**

- $\mathscr{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$,

**Section 7.3:**

- $\mathscr{L}\{e^{at} f(t)\} = \mathscr{L}\{f(t)\} \mid_{s \to s-a} = F(s-a)$, where $a \in \mathbb{R}$
- $\mathscr{L}^{-1}\{F(s-a)\} = \mathscr{L}^{-1}\{F(s) \mid_{s \to s-a}\} = e^{at} f(t)$
- $\mathscr{L}\{f(t-a)\mathscr{U}(t-a)\} = e^{-as}F(s)$, where $a > 0$
- $\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathscr{U}(t-a)$, where $a > 0$
- $\mathscr{L}\{g(t)\mathscr{U}(t-a)\} = e^{-as}\mathscr{L}\{g(t+a)\}$, where $a > 0$
- $\mathscr{L}\{\mathscr{U}(t-a)\} = \frac{e^{-as}}{s}$, where $a > 0$