Math 2C03: Assignment #3
Due: Friday, July 10th

McMaster University

Part I: Online

The online portion of your assignment is available on WeBWork.

Part II: Written

Please deposit the written part of this assignment in the course locker (basement of HH) by 2pm on the due date.

If you are unable to make it to campus, I will accept online submissions: either TeX your assignment OR *scan* your handwritten assignment. (Please do not take a photo, because it will be too difficult to read). Submit your file as a PDF via email, making the title your LastName_FirstName_Assignment2.

Questions:

Answer each question fully, explaining all reasoning. If you use a Theorem, explain why you are allowed to use it (i.e. why are the assumptions of the theorem satisfied?)

1. (4ts) Solutions of autonomous differential equations behave asymptotically (approach a value arbitrarily close) and have the translation property (if $y(x)$ is a solution of an autonomous DE $y' = f(y)$, then $y_1(x) = y(x-k)$ is also a solution $\forall k \in \mathbb{R}$). We briefly discussed why both of these things were true in class, but didn't formally write down the details. Please give a detailed explanation why both of these facts are true. In your argument, be sure to include why graphs of nonconstant solutions can't cross the graph of a constant solution, can't oscillate or have relative maximums/minimums, and why they must always be increasing or decreasing. (Recall that throughout Section 2.1, given an autonomous DE $y' = f(y)$, we assume that f and f' are continuous on some interval I.)

2. (3pts) Suppose that the population p (in thousands) of squirrels in Hamilton can be modelled by the differential equation $\frac{dp}{dt} = p(2-p)$.

   (a) If the initial population of squirrels is 3000, what can you say about the long-term behaviour of the squirrel population?

   (b) Can a population of 1000 ever decline to 500? Explain.

   (c) Can a population of 1000 ever increase to 3000? Explain.

3. (3pts) Consider the first-order differential equations

$$y' = (2-y)(3-y), \quad y' = (y-2)(3-y), \quad y' = (2-x)(3+x), \quad y' = (2-y)(3+x).$$

   Assign the direction fields below to the appropriate differential equation, and write a short paragraph to justify your choices.