Mat 202 - Tutorial #2

Recall: The number of nonnegative integer solutions to $x_1 + x_2 + \cdots + x_k = n$, $x_i \geq a_i$ is given by

$$\binom{n - a_1 - a_2 - \cdots - a_k + k - 1}{k-1} \quad (*)$$

i.e.) Selections of n objects from k types with at least a_i objects of type i is given by $(*)$.

1. How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ with $x_i \geq i$?

$n = 29$, $k = 6$, $a_i = i$.

$$\binom{29 - 1 - 2 - 3 - 4 - 5 - 6 + 6 - 1}{6-1} = \binom{13}{5} = 1287.$$

2. How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 40$?

$n = 40$, $k = 5$, $a_i = 0$.

$$\binom{40 + 5 - 1}{5-1} = \binom{44}{4}.$$

3. There are 5 types of color T-shirts on sale: black, blue, green, orange, & white. Alice is going to buy 10 T-shirts; she has to buy at least 2 blues, 2 oranges, & at least one for all other colors. Find the number of ways that Alice can select 10 T-shirts.

$x_1 \leftrightarrow$ # of black
$x_2 \leftrightarrow$ # of blue's
$x_3 \leftrightarrow$ # of green's
$x_4 \leftrightarrow$ # of orange
$x_5 \leftrightarrow$ # of white

Need $x_1+x_2+x_3+x_4+x_5 = 10$

$x_2 \geq 2,\ x_4 \geq 2,$
$x_1, x_3, x_5 \geq 1.$

$a_1 = a_3 = a_5 = 1,\ a_2 = a_4 = 2,\ n = 10,\ k = 5.$

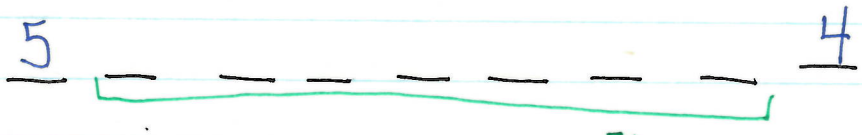
$$\binom{10-1-2-1-2-1+5-1}{5-1} = \binom{7}{4} = \frac{7!}{4!\,3!} = \frac{7\cdot 6\cdot 5}{6} = 35.$$

4. (a) In how many ways can the letters in the word CALCULATE be arranged.

9 letters. If they were all distinct there would be $9!$ ways.
However, there are 2 C's, 2 L's, 2 A's, and one of each other letter.

$$\frac{9!}{2!\,2!\,2!} \text{ ways.}$$

(b) What if we require that arrangement begins and ends with a consonant? Then how many ways?



2 C
2 L
T

2 A
4
E

5 consonants 4 vowels

$$\frac{7!(5)(4)}{2!2!2!}.$$

5. How many different arrangements of length 5 are there for the letters a, b, c, d, e if letters can be repeated?

$$\underline{5}\cdot\underline{5}\cdot\underline{5}\cdot\underline{5}\cdot\underline{5} = 5^5 = 3125.$$

6. Rose wants to arrange ornaments on her shelf. She has 2 identical cat ornaments, 3 identical dogs, 1 penguin, 1 rabbit, and 1 koala. How many way can she arrange the ornaments in a line on her shelf?

$$\frac{8!}{2!3!} \text{ ways.}$$