Mat 202 - Tutorial #11

Recall: If $P(B) \neq 0$, then the conditional probability of A given B is

$$P(A|B) := \frac{P(A \cap B)}{P(B)}.$$

Recall: Two events A and B are mutually exclusive if $A \cap B = \emptyset$.

Recall: Bayes' Formula: Let B_1 and B_2 be mutually exclusive events s.t. $P(B_1) + P(B_2) = 1$. Then

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}.$$

1. The probability that a flight departs on time is 0.83; the probability that it arrives on time is 0.82; the probability that it departs and arrives on time is 0.78. Find the probability that the plane

a) Arrives on time given that it departed on time.

$P(D) = 0.83$. $P(A) = 0.82$. $P(D \cap A) = 0.78$.

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.78}{0.83} = \frac{78}{83} \approx 0.94.$$

b) Departed on time given that it arrived on time.

$$P(D|A) = \frac{0.78}{0.82} = \frac{78}{82} \approx 0.95.$$

c Arrives late given that it departed on time.

$$P(A^c \mid D) = \frac{P(A^c \cap D)}{P(D)}.$$

Note that $P(A \cap D) + P(A^c \cap D) = P(D)$. So,

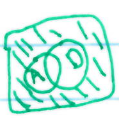
$$P(A^c \mid D) = \frac{P(D) - P(A \cap D)}{P(D)} = 1 - \frac{P(A \cap D)}{P(D)} = 1 - P(A \mid D)$$

$$= 1 - \frac{78}{83} = \frac{5}{83} \approx 0.06.$$

d Departed late given that it arrived on time.

$$P(D^c \mid A) = 1 - P(D \mid A) = 1 - \frac{78}{82} = \frac{4}{82} = \frac{2}{41} \approx 0.05.$$

e Arrives late given that it departed late.



$$P(A^c \mid D^c) = \frac{P(A^c \cap D^c)}{P(D^c)}.$$

Note that $A^c \cap D^c = (A \cup D)^c$ and $P(A \cup D) = P(A) + P(D) - P(A \cap D)$.

$$\therefore P(A^c \mid D^c) = \frac{1 - P(A) - P(D) + P(A \cap D)}{1 - P(D)}$$

$$= \frac{1 - 0.82 - 0.83 + 0.78}{1 - 0.83} = \frac{0.13}{0.17} = \frac{13}{17} \approx 0.76.$$

2. [9.10]: We roll 2 dice, one red and one green. Under each assumption below, what is the probability that the roll is double-sixes?

Red Dice

Green Dice	1	2	3	4	5	6
1	(1,1)	(2,1)				
2	(1,2)					
3	(1,3)					
4	⋮					
5	⋮					
6	etc.					(6,6)

a The red die shows a 6.

Let R_6 be the event that the red die shows a 6. $P(R_6) = 1/6$.

Let D be the event that both dice show a 6. We must have that $D \subseteq R_6$ (since if both dice show a 6 then the red die must too) $\Rightarrow P(D \cap R_6) = P(D)$. We know $P(D) = 1/36$.

So, $P(D|R_6) = \dfrac{P(D \cap R_6)}{P(R_6)} = \dfrac{P(D)}{P(R_6)} = \dfrac{1/36}{1/6} = \dfrac{1}{6}$.

b At least one dice shows a six.

Do we roll both dice at the same time or one at a time?

<u>Method 1</u>: If we roll the red die then the green one:

We're in the same situation as **a**. $P(D|R_6) = 1/6$.

<u>Method 2</u>: we roll both at same time.

Let E denote the event that at least one dice shows a 6. Then $P(E) = 11/36$. We have $D \subseteq E$, so $D \cap E = D$.

$$P(D|E) = \frac{P(D \cap E)}{P(E)} = \frac{P(D)}{P(E)} = \frac{1/36}{11/36} = \frac{1}{11}.$$

3. Marie is getting married tomorrow. It's an outdoor wedding. In recent years, it has only rained 5 days each year. However, the weatherman predicted rain for tomorrow. When it actually rains, the weatherman correctly forcasts rain 90% of the time. When it doesn't rain, he incorrectly forcasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Let R be the event that it rains and R^c the event that it doesn't rain. Note that $P(R) + P(R^c) = 1$. Let W denote the event that the weatherman predicts rain.

We want to find $P(R|W)$.

We Know that $P(W|R) = 0.9$ and $P(W|R^c) = 0.1$.

We also Know that $P(R) = \frac{5}{365}$ and $P(R^c) = \frac{360}{365}$.

$$P(R|W) = \frac{P(W|R)\,P(R)}{P(W|R)\,P(R) + P(W|R^c)\,P(R^c)} = \frac{\frac{9}{10}\cdot\frac{5}{365}}{\frac{9}{10}\cdot\frac{5}{365} + \frac{1}{10}\cdot\frac{360}{365}}$$

$\begin{array}{r}81\\5\overline{)405}\end{array}$

$$= \frac{\frac{45}{3650}}{\frac{45+360}{3650}} = \frac{45}{405} = \frac{9}{81} = \frac{1}{9}.$$

$\therefore$ The probability that it will rain is $\frac{1}{9} \approx 0.11$.

(O.J. was on trial for killing his wife).

4. In the O.J. Simpson trial, it was known that O.J. abused his wife. His defense tried to belittle this evidence by citing the statistic: "only 1/1000 abusive husbands eventually murder their wife." Is this a logical argument?

In this statistic, we're ignoring the fact that OJ's wife <u>was</u> murdered.

So, we want the probability that a man murders his wife given that he previously battered her <u>and</u> she was murdered by someone.

Let B be the event that a husband batters his wife.
Let M be the event that the wife is murdered.
Let G be the event that a husband murders his wife.

We want to find $P(G \mid B \cap M)$.

In 1994, in a population of 100,000,000 women, 5000 were murdered and 1500 by their husband.

So, $P(M \mid G^c) = \frac{3500}{100{,}000{,}000} = \frac{7}{20{,}000{,}000}$.
We also know $P(G \mid B) = 1/1000$.

$$P(G \mid B \cap M) = \frac{P(G \cap B \cap M)}{P(B \cap M)} = \frac{\frac{P(M \cap G \cap B)}{P(G \cap B)} \, \frac{P(G \cap B)}{P(B)} \, P(B)}{P(M \cap B)}$$

$$= \frac{P(M \mid G \cap B) P(G \mid B) P(B)}{P(B \cap M)} = \frac{P(M \mid G \cap B) P(G \mid B) \cancel{P(B)}}{P(M \mid B) \cancel{P(B)}}$$

$$= \frac{P(M \mid G \cap B)\, P(G \mid B)}{P(M \mid B)}.$$

Note that $P(M \mid B \cap G)\, P(G \mid B) + P(M \mid B \cap G^c)\, P(G^c \mid B)$

$$= \frac{P(M \cap B \cap G)}{\cancel{P(B \cap G)}} \frac{\cancel{P(G \cap B)}}{P(B)} + \frac{P(M \cap B \cap G^c)}{\cancel{P(B \cap G^c)}} \frac{\cancel{P(G^c \cap B)}}{P(B)}$$

$$= \frac{P(M \cap B)}{P(B)} = P(M \mid B).$$

So, $$P(G \mid M \cap B) = \frac{\overbrace{P(M \mid G \cap B)}^{=1}\, \overbrace{P(G \mid B)}^{=1/1000}}{\underbrace{P(M \mid B \cap G)}_{=1}\, \underbrace{P(G \mid B)}_{=1/1000} + \underbrace{P(M \mid B \cap G^c)}_{7/20{,}000{,}000}\, \underbrace{P(G^c \mid B)}_{999/1000}}$$

$$\approx 0.9997.$$