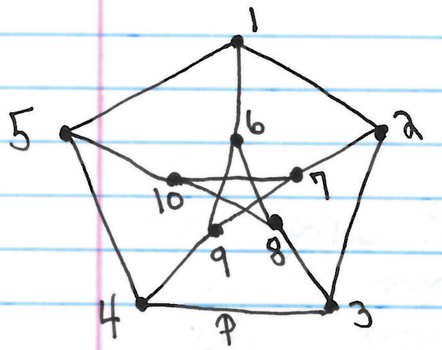
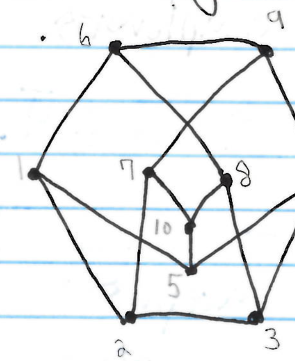
Mat 202 - Tutorial #10

1. Prove that the following graphs are isomorphic.



Peterson Graph



B

1 2
1 5
1 6
2 3
2 7
3 4
3 8
4 5
4 9
5 10
6 8
6 9
7 9
7 10
8 10

Each edge pair matches in each graph.

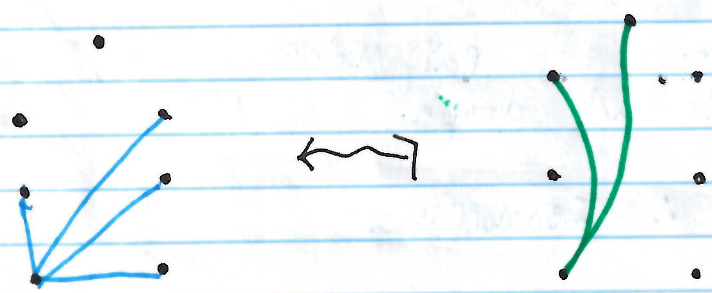
∴ Isomorphic.

2. [11.15] Prove that there are exactly 2 isomorphism classes of 7-vertex simple graphs in which every vertex has degree 4.

<u>Hint</u>: Consider the complements.

<u>Recall</u>: $G \cong H \iff \bar{G} \cong \bar{H}$.

$\therefore$ It suffices to show that there are exactly 2 isomorphism classes of 7 vertex graphs in which every vertex has degree 2.



Let's denote our original graph by G and its complement by $\bar{G}$.

$\bar{G}$ has 7 vertices & each has degree 2

$\Rightarrow$ # edges in $\bar{G} = \frac{1}{2}(7 \cdot 2) = 7$.

Every connected component of $\bar{G}$ must be a cycle (b/c pick a vertex v_0. v_0 has 2 neighbors. Pick one of them and call it v_1. v_1 has another neighbor $v_2 \neq v_0$ (b/c $\bar{G}$ simple). Suppose you continue in this way and find a path $v_0 v_1 \cdots v_k$. (If this is not possible then it's a cycle). Then v_k can't be adjacent to $v_1, \ldots, v_{k-1}$ b/c each already have deg. 2. If it's adjacent to $v_{k+1} \neq v_0$ then this path will keep getting longer until you run out of vertices &

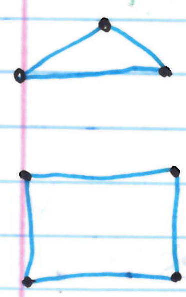
then the last vertex in the chain must be adjacent to v_0).

Now how many ways can be partition the vertices of $\bar{G}$ into cycles?

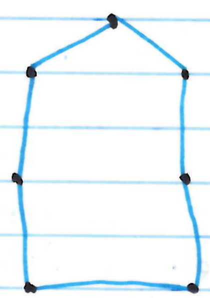
Can have cycles of length 3, 4, 5, 6, 7. So how many ways to add these #'s to make 7?

$\left.\begin{array}{l} 3+4=7 \\ 7 \end{array}\right\}$ only 2 ways.

So, $\bar{G}$ has the form:



or



and hence has 2 equivalency classes and so G also has exactly 2 equivalency classes.

3. If G is a planar connected simple graph with at least 2 edges, prove that the degree of G is greater than or equal to 3 times the number of faces in G.

$\deg G := \sum_{v \in V(G)} \deg(v)$.

<u>Recall</u>: <u>Euler's Formula</u>: $V(G) - E(G) + F(G) = 2$ if G is a connected planar graph.

We know $\deg G = \frac{1}{2} E(G)$.

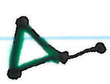
If G has exactly one face then it must look like:

A path of length at least 3

(with possibly some other isolated vertices).

$F(G) = 1$. $\deg G \geq 4$. $4 \geq 3 \cdot 1$ ✓

If G has more than one face $\Rightarrow$ each face in G is bounded by at least 3 edges.



sum boundary edges of each face $\geq 3F$

$2E(G)$ (b/c edge in G counted at most twice)

So, $2E(G) \geq 3F$. Since $\deg G = 2E(G)$ we have $\deg G \geq F(G)$ as desired.