Office Hours:
Mon.: 3-5pm
[starting Sept. 21st].

Website:
ms.mcmaster.ca/~dedieula/2203.html.

Math 2203 - Tutorial #1

- ODE vs. PDE: ODE has a single independent variable, whereas a PDE has at least two independent variables.

e.g.) $\frac{d^2y}{dx^2} + 7y - \left(\frac{dy}{dx}\right)^2 = 8$ [O.D.E.]

$\frac{d^2y}{dx^2} + \frac{dy}{dt} = 8$ [PDE]

- Order: highest derivative in eq^n.

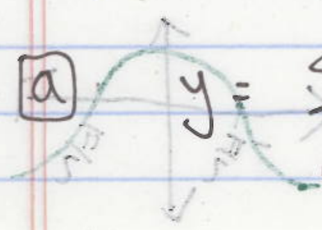
1. State the order of the following D.E. Linear?

$\boxed{a}$ $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$. Order 2.
Not linear b/c $\left(\frac{dy}{dx}\right)^3$ under $\sqrt{\ }$.

$\boxed{b}$ $(\sin t)y''' - (\cos t)y' = 10$. order 3. Linear.

2. Is the piece-wise defined function a solution to the given DE on the interval provided? Explain.

$\boxed{a}$ $y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$, $xy' - 2y = 0$, $I = (-\infty, \infty)$.



Solution to #2 on page 3.

$\boxed{b}$ $y = \begin{cases} \sqrt{25 - x^2}, & -5 < x < 0 \\ -\sqrt{25 - x^2}, & 0 \leq x < 5 \end{cases}$

3. Given that $y = \sin x$ is an explicit solution of $y' = \sqrt{1 - y^2}$, Find an interval of def^{n}.

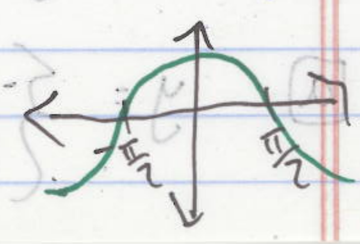
We need to Find an interval I s.t. $y = \sin x$ is C^1 on I AND when $y = \sin x$ is plugged into the eq^{n} $y' = \sqrt{1 - y^2}$, it reduces it to the identity.

$y = \sin x$ is C^1 everywhere, since $y' = \cos x$ cont. everywhere.

Plugging $y = \sin x$ into the eq^{n}:

$y' = \sqrt{1 - y^2} \iff \cos x = \sqrt{1 - \sin^2 x} \iff \cos x = \sqrt{\cos^2 x}$

$\iff \cos x = |\cos x|$. This is only true for those values of x where $\cos x$ is positive. So we can choose any I where $\cos x \geq 0$. In particular, $I = (-\frac{\pi}{2}, \frac{\pi}{2})$ does the job.



2.a) Verify that the piece-wise-defined function

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$ is a solution of the DE

$xy' - 2y = 0$ on $(-\infty, \infty)$.

The DE is 1^{st}-order, so must show y satisfies DE & is C^1 on $(-\infty, \infty)$.

$\underline{x<0}$: $\frac{d}{dx}(-x^2) = -2x$. $xy' - 2y = x(-2x) - 2(-x^2) = -2x^2 + 2x^2 = 0$. ✓

$\underline{x \geq 0}$: $\frac{d}{dx}(x^2) = 2x$. $xy' - 2y = x(2x) - 2(x^2) = 0$. ✓

Need to check if y' continuous on $(-\infty, \infty)$.

$$y' = \begin{cases} -2x, & x < 0 \\ 2x, & x \geq 0. \end{cases} \quad \lim_{x \to 0^-} y' = \lim_{x \to 0} -2x = 0.$$

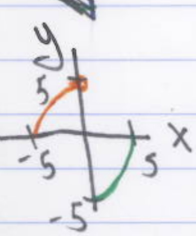
same!

$$\lim_{x \to 0^+} y' = \lim_{x \to 0} 2x = 0.$$

$\therefore$ DE C^1 on $(-\infty, \infty) \Rightarrow y$ is a solution on $(-\infty, \infty)$.

b) In Ex. 5 we saw $y = \sqrt{25 - x^2}$ & $y = -\sqrt{25 - x^2}$ are solutions of $y' = -\frac{x}{y}$ on $(-5, 5)$. Explain why

$$y = \begin{cases} \sqrt{25 - x^2} & -5 < x < 0 \\ -\sqrt{25 - x^2} & 0 \leq x < 5 \end{cases}$$ not a solution to DE on $(-5, 5)$.



We can see y is not cont. @ 0:

$\lim_{x \to 0^-} y = 5$.

not same

$\lim_{x \to 0^+} y = -5$.

4. Determine whether the Existence/Uniqueness Theorem guarantees that the DE $y' = \sqrt{y^2-9}$ possesses a unique solution through the given point.

$\boxed{a}$ $(1,4)$, $\boxed{b}$ $(2,-3)$.

Here $F(x,y) = \sqrt{y^2-9}$. The Theorem is satisfied for points inside (in the interior) of a region R, where F & $\frac{\partial F}{\partial y}$ are cont. on R.

$F(x,y) = \sqrt{y^2-9}$ is cont. so long as $y^2-9 \ge 0$

$\iff y^2 \ge 9 \iff |y| \ge 3 \iff y \ge 3$ or $y \le -3$

$\frac{\partial F}{\partial y} = \frac{y}{\sqrt{y^2-9}}$ is cont $\iff y^2-9 > 0$

$\iff y > 3$ or $y < -3$.

So, we could choose any points (x,y) s.t. y lies in the interval $(-\infty,-3)$ or $(3,\infty)$.

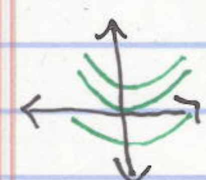
$\boxed{a}$: $4 \in (3,\infty)$, so Theorem satisfied for $(1,4)$. ✓

$\boxed{b}$ $-3 \notin (-\infty,-3)$ & $-3 \notin (3,\infty)$, so Theorem not satisfied for $(2,-3)$.

5. Suppose you're given a 1^{st} order DE $y' = F(x,y)$, where F & $\frac{\partial F}{\partial y}$ are cont. on a rectangular region R.

Could 2 solution curves in its 1-par. family of solutions intersect at a point in R? Why or why not?

e.g. 1-par. family: $y = x^2 + c$



F & $\frac{\partial F}{\partial y}$ cont. on $R \Rightarrow$ For any point (x_0, y_0) in R, a solution exists & is unique in some neighborhood around that point (by Existence / Uniqueness Theorem). Therefore, in that nbhd 2 solutions can't intersect b/c there's a <u>unique</u> solution in that nbhd.
This is true for all pts in R.
$\therefore$ 2 curves can't intersect in R.

6. Match the DE to the direction field. [see slides].

$\boxed{a}$ $y' = (2-y)(3-y)$, $\boxed{b}$ $y' = (y-2)(3-y)$, $\boxed{c}$ $y' = (2-x)(3+x)$,
$\boxed{d}$ $y' = (2-y)(3+x)$.

<u>Looking at 1^{st} Direction Field</u>: $y' = 0$ at $y = 2$ & $y = 3$
$\Rightarrow$ could be $\boxed{a}$ or $\boxed{b}$.
When $2 < y < 3$, $y' > 0 \Rightarrow$ it must be $\boxed{b}$.

$\boxed{1} \Leftrightarrow \boxed{b}$.

3rd Direction Field: Also $y' = 0$ at $y = 2$ & $y = 3$ & for $2 < y < 3$ we have $y' < 0$ $\Rightarrow$ corr. to $\boxed{a}$. $\boxed{3} \Leftrightarrow \boxed{a}$.

2nd Direction Field: $y' = 0$ at $x = 2$ & $x = -3$ $\Rightarrow$ corr. to $\boxed{c}$.

$\boxed{2} \Leftrightarrow \boxed{c}$.

[$\boxed{d}$ doesn't corr. to anything].