# Math 2X03: Week #11 Practice Problems

5.3: #1, 3 ⓒⓓ, 9, 11, 13, 15, 19.
5.4: #1 ⓐⓑ, 3 ⓐⓒ, 5, 9, 11, 15, 17, 19

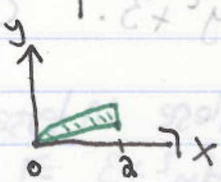
**5.3:**

1. Match the iterated integral to the correct region of integration.

ⓐ $\int_1^2 \int_{\ln x}^{e^x} dy\,dx$

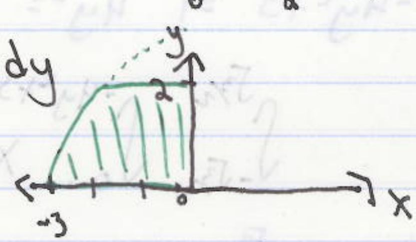

iii

ⓑ $\int_0^2 \int_{\frac{1}{8}x}^{x^{1/3}} dy\,dx$




iv

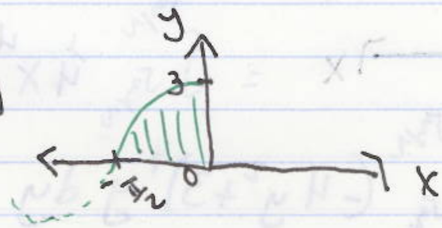
ⓒ $\int_0^2 \int_{-\sqrt{9-y^2}}^{0} dx\,dy$




ii

X-simple: IF cont. functions $\psi_1$ & $\psi_2$ on $y\in[c,d]$ s.t. $\psi_1(y) \le x \le \psi_2(y)$.

ⓓ $\int_0^3 \int_{\arccos \frac{y}{3}}^{0} dx\,dy$




i

$\boxed{3.}$ Evaluate, draw region D, & state whether x-simple, y-simple, or simple.

ⓒ $\int_0^1 \int_1^{e^x} (x+y)\,dy\,dx = \int_0^1 xy + \frac{y^2}{2} \Big|_1^{e^x} dx$

For y-simple: $x\in[0,1]$ & $1\le y\le e^x$. For x-simple: $y\in[1,e]$, $\ln y \le x \le 1$.

$u=2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$= \int_0^1 xe^x + \frac{1}{2}e^{2x} - [x + \frac{1}{2}]\,dx$


Simple (i.e. both x & y simple)

$= \int_0^1 xe^x\,dx + \int_0^1 (\frac{1}{2}e^{2x} - x - \frac{1}{2})\,dx$

$= xe^x - \int e^x dx + \frac{1}{4}e^{2x} - \frac{x^2}{2} - \frac{1}{2}x$

$u=x \quad v=e^x$
$du=dx \quad dv=e^x$

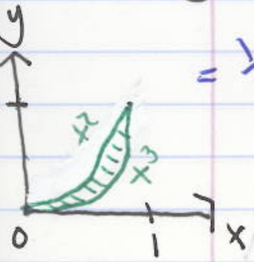
$= xe^x - e^x + \frac{1}{4}e^{2x} - \frac{x^2}{2} - \frac{1}{2}x \Big|_0^1 = e^1 - e^1 + \frac{1}{4}e^2 - \frac{1}{2} - \frac{1}{2} - [-1 + \frac{1}{4}]$

$= \frac{1}{4}e^2 - 1 + 1 - \frac{1}{4} = \frac{1}{4}e^2 - \frac{1}{4} = \frac{1}{4}(e^2-1).$

(d) $\int_0^1 \int_{x^3}^{x^2} y \, dy\, dx = \int_0^1 \frac{1}{2}y^2 \Big|_{x^3}^{x^2} dx = \int_0^1 \frac{1}{2}x^4 - \frac{1}{2}x^6 \, dx$

$= \frac{1}{2}\left[\frac{1}{5}x^5 - \frac{1}{7}x^7\right]\Big|_0^1 = \frac{1}{2}\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{1}{2}\left[\frac{7}{35} - \frac{5}{35}\right]$

$= \frac{1}{2}\left[\frac{2}{35}\right] = \boxed{\frac{1}{35}}.$




simple

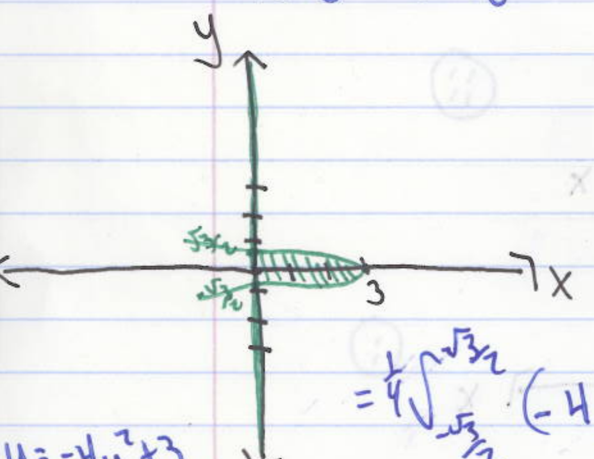
For y-simple: $x \in [0,1]$ & $x^3 \le y \le x^2$.

For x-simple: $y \in [0,1]$ & $\sqrt{y} \le x \le y^{1/3}$.

$y = x^2 \Rightarrow x = \sqrt{y}$. $y = x^3 \Rightarrow x = y^{1/3}$.

$\boxed{9.}$ Let $D$ be the region bounded by the y-axis & the parabola $x = -4y^2 + 3$. Compute $\iint_D x^3 y \, dx\, dy$.

Let's see where these intersect: on the y-axis we have $x = 0 \Rightarrow 0 = -4y^2 + 3 \Rightarrow 4y^2 = 3 \Rightarrow y^2 = 3/4 \Rightarrow y = \pm\frac{\sqrt{3}}{2}$.




$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} \int_0^{-4y^2+3} x^3 y \, dx\, dy$

$= \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{1}{4}x^4 y \Big|_0^{-4y^2+3} dy$

$= \frac{1}{4}\int_{-\sqrt{3}/2}^{\sqrt{3}/2} (-4y^2+3)^4 y \, dy$

$u = -4y^2 + 3$

$du = -8y\,dy$

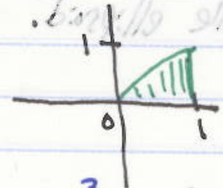
$-\frac{1}{8}du = y\,dy$

$= \frac{1}{4} \cdot -\frac{1}{8} \int u^4 \, du = \frac{-1}{32} \frac{1}{5} u^5 \Big| = \frac{-1}{32} \cdot \frac{1}{5}\left[-4y^2+3\right]\Big|_{-\sqrt{3}/2}^{\sqrt{3}/2}$

$= \frac{-1}{32} \cdot \frac{1}{5}\left[0 - 0\right] = \boxed{0}.$

$\boxed{5.}$ Change the order of integration & evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy$$




$x = \sqrt{y}$
$\Rightarrow y = x^2.$

$$= \int_0^1 \int_0^{x^2} e^{x^3}\, dy\, dx = \int_0^1 y e^{x^3} \Big|_0^{x^2} dx$$

$u = x^3$
$du = 3x^2\, dx$
$\frac{1}{3} du = x^2\, dx$

$$= \int_0^1 x^2 e^{x^3}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3} e^{x^3} \Big|_0^1 = \frac{1}{3}\left[e^1 - 1\right].$$