

Def'n: $F: U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$.

The level set of value c is $\{x \in U \mid F(x) = c\}$.

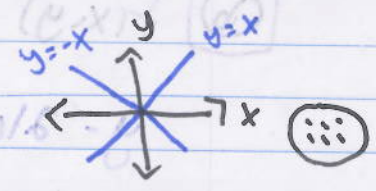
If $n=2$ it's called a level curve.

If $n=3$ it's a level surface.

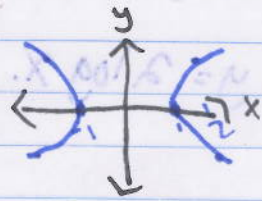
Match the given level curves with their visual descriptions.

① $F(x,y) = x^2 - y^2 = c$

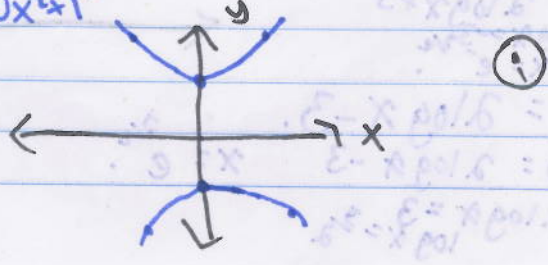
$c=0: x^2 - y^2 = 0 \Rightarrow y^2 = x^2 \Rightarrow y = \pm x$



$c=1: x^2 - y^2 = 1 \Rightarrow y^2 = x^2 - 1 \Rightarrow y = \pm \sqrt{x^2 - 1}$



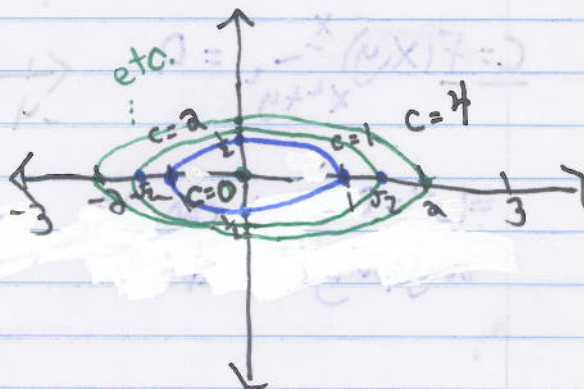
$c=-1: x^2 - y^2 = -1 \Rightarrow y^2 = x^2 + 1 \Rightarrow y = \pm \sqrt{x^2 + 1}$



7. Sketch the level curves & graphs of the following functions:

⑥ $F: \mathbb{R}^2 \rightarrow \mathbb{R}$
 $(x,y) \mapsto x^2 + 4y^2$

$x^2 + 4y^2 = c$



$c=0: \exists x=y=0$

We know $x^2 + 4y^2 \geq 0$, so we know $c \geq 0$.

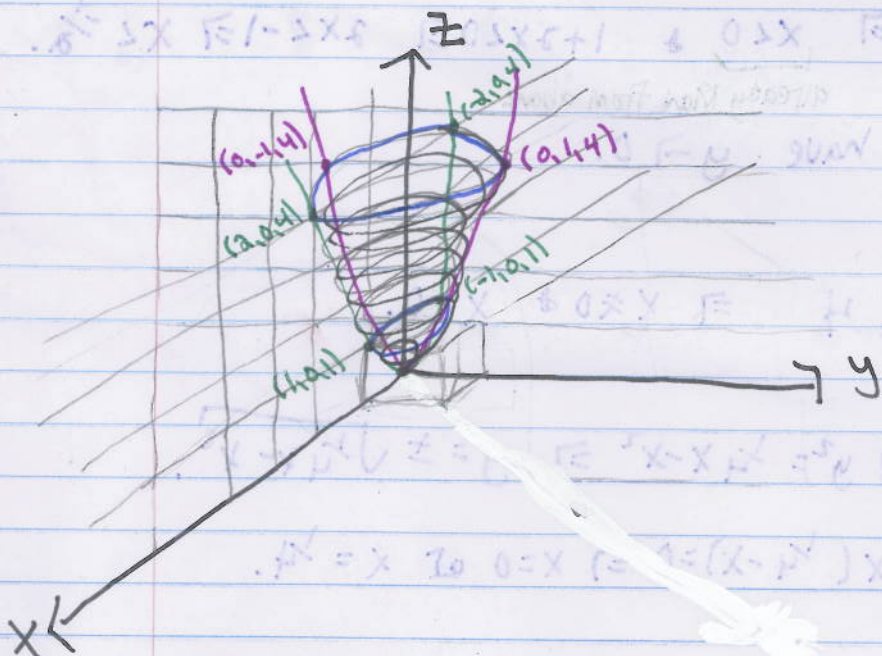
$c=1: x^2 + 4y^2 = 1 \Rightarrow x^2 + \left(\frac{y}{\frac{1}{2}}\right)^2 = 1$

$c=2: x^2 + 4y^2 = 2 \Rightarrow x^2 + 2y^2 = 1 \Rightarrow \left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{\frac{1}{\sqrt{2}}}\right)^2 = 1$

$c=4: x^2 + 4y^2 = 4 \Rightarrow \left(\frac{x}{2}\right)^2 + y^2 = 1$

Section $y=0$: $z = x^2$

Section $x=0$: $z = 4y^2$



13. Draw the level curves (in the xy -plane) for the given function F at values c . Sketch the graph of $z = F(x, y)$.

$$F(x, y) = \sqrt{100 - x^2 - y^2}, \quad c = 0, 2, 4, 6, 8, 10.$$

$$c=0: \sqrt{100 - x^2 - y^2} = 0 \Rightarrow 100 - x^2 - y^2 = 0 \Rightarrow x^2 + y^2 = 100.$$

$$c=2: \sqrt{100 - x^2 - y^2} = 2 \Rightarrow 100 - x^2 - y^2 = 4 \Rightarrow x^2 + y^2 = 96. \quad (\sqrt{96})$$

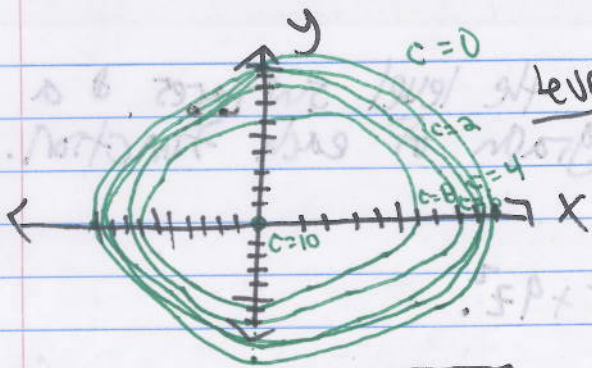
$$c=4: x^2 + y^2 = 100 - 16 \Rightarrow x^2 + y^2 = 84. \quad (\sqrt{84} \approx 9.165)$$

$$c=6: x^2 + y^2 = 64$$

$$c=10: x^2 + y^2 = 0 \Rightarrow$$

$$x = y = 0.$$

$$c=8: x^2 + y^2 = 36$$



Level Curves.

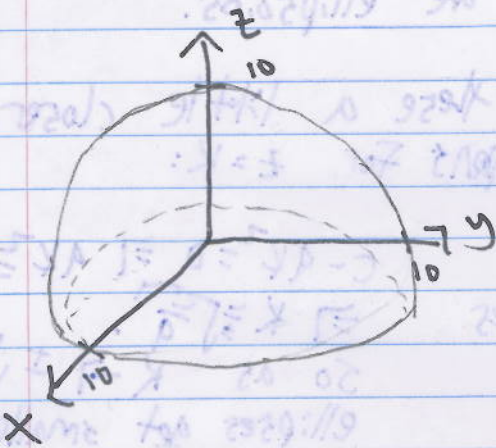
$$z = \sqrt{100 - x^2 - y^2}$$

$$\Leftrightarrow z^2 = 100 - x^2 - y^2 \Rightarrow x^2 + y^2 + z^2 = 100 \quad (\text{Upper half of the sphere})$$

$$\& 100 - x^2 - y^2 \geq 0$$

$$\& 100 - x^2 - y^2 \geq 0$$

(Upper half of the sphere).



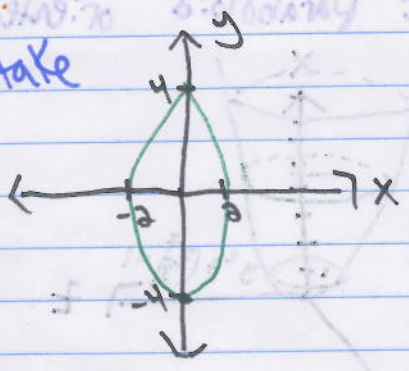
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Sketch or describe the surfaces in \mathbb{R}^3 of the eqⁿ:
21. $4x^2 + y^2 = 16$.

This eqⁿ doesn't depend on z , so it will be an ellipse $4x^2 + y^2 = 16$ for all values of z .

i.e. we take



and stretch it up & down the z -axis to get a cylinder.

