

## Math 2C03 - Assignment #1

1. Suppose  $x = e^{zt}$ .

(a) Find value of  $x''' - 6x'' + 12x' - 8x$  in terms of  $t$ .

$$x' = ze^{zt} \quad x'' = z^2 e^{zt} \quad x''' = z^3 e^{zt}$$

$$x''' - 6x'' + 12x' - 8x = z^3 e^{zt} - 6z^2 e^{zt} + 12z e^{zt} - 8e^{zt}$$

(b) Simplify and enter a DE in terms of  $x$  satisfied by  $x = e^{zt}$ .

$$x''' - 6x'' + 12x' - 8x = 0.$$

(c) Is  $x = e^t$  a solution?

$$\text{No, b/c } e^t - 6e^t + 12e^t - 8e^t = -e^t \neq 0.$$

2. Find a solution to  $\frac{dA}{dt} = -2A$ , if  $A(0) = 3$ .

By inspection: if  $A = 3e^{-2t}$ , then  $\frac{dA}{dt} = -6e^{-2t} = -2A$ .

$\therefore$  A solution is  $A = 3e^{-2t}$  on  $(-\infty, \infty)$ .

3. Suppose  $x = Ae^{-t} + Be^{4t}$ .

(a) Verify that  $x = Ae^{-t} + Be^{4t}$  is a solution to  $x'' - 3x' - 4x = 0$ .

$$x' = -Ae^{-t} + 4Be^{4t} \quad x'' = Ae^{-t} + 16Be^{4t}$$

$$x'' - 3x' - 4x = Ae^{-t} + 16Be^{4t} + 3Ae^{-t} - 12Be^{4t} - 4Ae^{-t} - 4Be^{4t} = 0 \quad \checkmark$$

⑥ Find  $x(t)$  if  $x(0) = 2$  and  $x'(0) = 1$ .

$$\begin{cases} x(0) = 2 \Rightarrow 2 = A + B \\ x'(0) = 1 \Rightarrow 1 = -A + 4B \end{cases} \Rightarrow 3 = 5B \Rightarrow B = 3/5 \Rightarrow A = 2 - 3/5 = 7/5$$

$$\therefore x(t) = \frac{7}{5} e^{-t} + \frac{3}{5} e^{4t}$$

4. Find a non-constant solution  $x(t)$  to  $(x')^2 + x^2 = 25$ .

$$\begin{cases} x = 5 \cos t \\ x' = -5 \sin t \end{cases} \Rightarrow 25(\sin^2 t + \cos^2 t) = 25 \checkmark$$

$$\begin{aligned} \text{or } x &= 5 \sin t \\ x' &= 5 \cos t \end{aligned}$$

5. Match DE w/ solutions  $\text{A } y = e^{3x}, \text{ B } y = e^{-4x}, \text{ C } y = \frac{1}{x}, \text{ D } y = \cos x$ .

1.  $y'' + y = 0$  **D**

$$\text{If } y = \cos x \Rightarrow y' = -\sin x \Rightarrow y'' = -\cos x \Rightarrow y'' + y = 0.$$

2.  $2x^2 y'' + 3xy' = y$  **C**

$$\begin{aligned} \text{If } y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} \Rightarrow y'' = \frac{2}{x^3} \Rightarrow 2x^2 y'' + 3xy' \\ = 4 \frac{1}{x} - \frac{3}{x} = \frac{1}{x} = y. \checkmark \end{aligned}$$

3.  $y'' + 7y' + 12y = 0$  **B**

$$\begin{aligned} \text{If } y = e^{-4x} \Rightarrow y' = -4e^{-4x} \Rightarrow y'' = 16e^{-4x} \Rightarrow y'' + 7y' + 12y \\ = 16e^{-4x} - 28e^{-4x} + 12e^{-4x} = 0. \checkmark \end{aligned}$$

4.  $y'' - 7y' + 12y = 0$  **A**

$$\text{If } y = e^{3x} \Rightarrow y' = 3e^{3x} \Rightarrow y'' = 9e^{3x} \Rightarrow y'' - 7y' + 12y = 9e^{3x} - 21e^{3x} + 12e^{3x} = 0. \checkmark$$