

7.b: Systems of Linear DE's

#1, 3, 9: Use the Laplace Transform to solve the given system of linear DE's.

$$1. \begin{cases} \frac{dx}{dt} = -x + y \\ \frac{dy}{dt} = 2x \end{cases}, \quad x(0) = 0, y(0) = 1.$$

$$\begin{cases} \mathcal{L}\{x'\} = -\mathcal{L}\{x\} + \mathcal{L}\{y\} \\ \mathcal{L}\{y'\} = 2\mathcal{L}\{x\} \end{cases} \Rightarrow \begin{cases} [sX(s) - x(0)] = -X(s) + Y(s) \\ [sY(s) - y(0)] = 2X(s) \end{cases}$$

$$\Rightarrow \begin{cases} X(s)[s+1] - Y(s) = 0 \\ -2X(s) + sY(s) = 1 \end{cases} \Rightarrow \begin{bmatrix} s+1 & -1 \\ -2 & s \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} s & 1 \\ 2 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2 + s - 2} \begin{bmatrix} 1 \\ s+1 \end{bmatrix}$$

$$\Rightarrow X(s) = \frac{1}{(s+2)(s-1)} \quad \& \quad Y(s) = \frac{s+1}{(s+2)(s-1)}$$

$$1 = A(s+1) + B(s+2)$$

$$s+1 = A(s-1) + B(s+2)$$

$$\Rightarrow 1 = 3B \Rightarrow B = \frac{1}{3}$$

$$\Rightarrow 2 = 3B \Rightarrow B = \frac{2}{3}$$

$$\&\Rightarrow 1 = -3A \Rightarrow A = -\frac{1}{3}$$

$$\&\Rightarrow -1 = -3A \Rightarrow A = \frac{1}{3}$$

$$X(s) = -\frac{1}{3(s+2)} + \frac{1}{3(s-1)} \quad Y(s) = \frac{1}{3(s+2)} + \frac{2}{3(s-1)}$$

$$\Rightarrow X(t) = -\frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^{t}$$

$$y(t) = \frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{2}{3} \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = \frac{1}{3}e^{-2t} + \frac{2}{3}e^{t}$$

$$3. \begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = 5x - y \end{cases}, \quad x(0) = -1, y(0) = 2.$$

$$\begin{cases} [sX(s) - x_0] = X(s) - 2Y(s) \\ [sY(s) - y_0] = 5X(s) - Y(s) \end{cases}$$

$$\Rightarrow \begin{cases} X(s)[s-1] + 2Y(s) = -1 \\ -5X(s) + [s+1]Y(s) = 2 \end{cases} \Rightarrow \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} s-1 & 2 \\ -5 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & -2 \\ 5 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \frac{1}{s^2+9} \begin{bmatrix} -s-5 \\ 2s-7 \end{bmatrix}$$

$$\Rightarrow X(s) = \frac{-(s+5)}{s^2+9} \quad \& \quad Y(s) = \frac{2s-7}{s^2+9}$$

$$\Rightarrow x(t) = -\frac{1}{2} \mathcal{L}^{-1} \left\{ \frac{s}{s^2+9} \right\} + \frac{5}{3} \mathcal{L}^{-1} \left\{ \frac{3}{s^2+9} \right\} \quad \& \quad y(t) = 2 \mathcal{L}^{-1} \left\{ \frac{s}{s^2+9} \right\} - \frac{7}{3} \mathcal{L}^{-1} \left\{ \frac{3}{s^2+9} \right\}$$

$$\Rightarrow x(t) = -\cos(3t) + \frac{5}{3} \sin(3t) \quad \& \quad y(t) = 2\cos(3t) - \frac{7}{3} \sin(3t).$$

$$9. \begin{cases} x'' + y'' = t^2 \\ x'' - y'' = 4t \end{cases}, \quad x(0) = 8, x'(0) = 0, \\ y(0) = 0, y'(0) = 0.$$

$$\begin{cases} [s^2X(s) - sx(0) - x'(0)] + [s^2Y(s) - sy(0) - y'(0)] = \frac{t^2}{s^3} \\ [s^2X(s) - sx(0) - x'(0)] - [s^2Y(s) - sy(0) - y'(0)] = \frac{4t}{s^2} \end{cases}$$

$$\Rightarrow \begin{cases} s^2X(s) + s^2Y(s) = \frac{2}{s^3} + 8s \\ s^2X(s) - s^2Y(s) = \frac{4}{s^2} + 8s \end{cases}$$

$$2s^2X(s) = \frac{2}{s^3} + \frac{4}{s^2} + 16s \quad \& \quad Y(s) = \frac{2}{s^3} - \frac{4}{s^2}$$

$$\Rightarrow X(s) = \frac{1}{s^5} + \frac{2}{s^4} + \frac{8}{s} \quad \Rightarrow Y(s) = \frac{1}{s^5} - \frac{2}{s^4}$$

$$\Rightarrow x(t) = \frac{1}{4!} t^4 + \frac{2}{3!} t^3 + 8 \text{ and } y(t) = \frac{1}{4!} t^4 - \frac{2}{3!} t^3$$

1. Find the interval of Power Series

5, 11, 17, 23, 29, 31

3. Find the interval & radius of convergence for $\sum_{n=0}^{\infty} \frac{t^n}{n!}$

Ratio Test:	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right $	$\lim_{n \rightarrow \infty} \left \frac{t^{n+1}/(n+1)!}{t^n/n!} \right $	$\lim_{n \rightarrow \infty} \left \frac{t}{n+1} \right = 0$
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$\rho = 0$ for all values of t . The series converges for all t .

Interval of convergence: $(-\infty, \infty)$

Radius of convergence: ∞

Interval of convergence: $(-\infty, \infty)$

The series converges for all values of t . The interval of convergence is $(-\infty, \infty)$.

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} = e^t$$

Interval of convergence: $(-\infty, \infty)$