Ch.7: The Laplace Transform

7.1: Def^{in} of the Laplace Transform

#1, 9, ~~13~~, 21, 29, 52, 5~~3~~

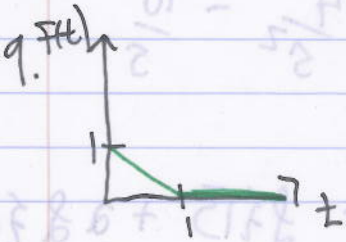
#1, 9, 13: Use the def^{in} of the Laplace transform to find $\mathcal{L}\{f(t)\}$.

1. $f(t) = \begin{cases} -1, & 0 \le t < 1 \\ 1, & t \ge 1. \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st}(-1)\,dt + \int_1^\infty e^{-st}\,dt$$

$$= \frac{e^{-st}}{s}\Big|_0^1 - \frac{e^{-st}}{s}\Big|_1^\infty = \frac{e^{-s}}{s} - \frac{1}{s} - 0 + \frac{e^{-s}}{s} = \frac{-1}{s} + \frac{2e^{-s}}{s}.$$

9.



So, $f(t) = \begin{cases} -t+1, & 0 \le t < 1 \\ 0, & t \ge 1. \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(-t+1)\,dt + \int_1^\infty e^{-st}(0)\,dt$$

$$= -(1-t)\frac{e^{-st}}{s}\Big|_0^1 - \int_0^1 \frac{e^{-st}}{s}\,dt$$

$u = 1-t$
$du = -dt$
$v = \frac{-e^{-st}}{s}$
$dv = e^{-st}$

$$= 0 + \frac{1}{s} + \left[\frac{e^{-st}}{s^2}\right]\Big|_0^1 = \frac{1}{s} + \frac{e^{-s}}{s^2} - \frac{1}{s^2}.$$

$=$

21, 29. Find $\mathcal{L}\{F(t)\}$ using Laplace Transform table.

$= \lim_{t \to \infty} \frac{1}{-(4-s)e^{-(4-s)t}}$

$= 0$ when $-(4-s) > 0$

i.e. when $s > 4$.

21. $F(t) = 4t - 10$.

$$\mathcal{L}\{F(t)\} = 4\mathcal{L}\{t\} - 10\mathcal{L}\{1\} = \frac{4}{s^2} - \frac{10}{s}.$$

29. $F(t) = (1+e^{2t})^2$.

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{1 + 2e^{2t} + e^{4t}\} = \mathcal{L}\{1\} + 2\mathcal{L}\{e^{2t}\} + \mathcal{L}\{e^{4t}\}$$

$$= \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}.$$

52. Explain why the function $F(t) = \begin{cases} t, & 0 \le t < 2 \\ 4, & 2 < t < 5 \\ \frac{1}{t-5}, & t > 5 \end{cases}$ is not piecewise cont. on $[0, \infty)$.

$$\lim_{t \to 5^+} F(t) = \lim_{t \to 5} \frac{1}{t-5} = \infty$$ $\Rightarrow$ on interval $(5, 6)$, 5 does not have finite right limit $\Rightarrow$ not piecewise cont. on $[0, \infty)$.