4.3: Homogeneous Linear Eq'n's w/ Constant Coefficients

#1, 5, 11, 15, 23, 35, 49, 55

#1, 5, 11, 15, 23 : Find general solution.

1. $4y'' + y' = 0.$

$4m^2 + m = 0 \Leftrightarrow m(4m+1) = 0 \Leftrightarrow m = 0$ or $m = -\frac{1}{4}.$

$y = c_1 + c_2 e^{-x/4}.$

5. $y'' + 8y' + 16y = 0.$

$m^2 + 8m + 16 = 0 \Leftrightarrow (m+4)^2 = 0 \Leftrightarrow m = -4.$

$y = c_1 e^{-4x} + c_2 x e^{-4x}.$

11. $y'' - 4y' + 5y = 0.$

$m^2 - 4m + 5 = 0 \Leftrightarrow m = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm i.$ Here $\alpha = 2$, $\beta = 1.$

$y = e^{2x}(c_1 \cos x + c_2 \sin x).$

15. $y''' - 4y'' - 5y' = 0.$

$m^3 - 4m^2 - 5m = 0 \Leftrightarrow m(m^2 - 4m - 5) = 0 \Leftrightarrow m(m-5)(m+1) = 0.$

$y = c_1 + c_2 e^{5x} + c_3 e^{-x}.$

23. $y^{(4)} + y''' + y'' = 0.$

$m^4 + m^3 + m^2 = 0 \Leftrightarrow m^2(m^2+m+1) = 0.$

$m = \frac{-1 \pm \sqrt{1-4}}{2} = \underbrace{-\frac{1}{2}}_{\alpha} \pm \underbrace{\frac{\sqrt{3}}{2}}_{\beta} i.$

$y = c_1 + c_2 x + e^{-\frac{x}{2}}\left[c_3 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_4 \sin\left(\frac{\sqrt{3}}{2}x\right)\right].$

35. $y''' + 12y'' + 36y' = 0,\ y(0)=0,\ y'(0)=1,\ y''(0)=-7.$
Solve this IVP.

$m^3 + 12m^2 + 36m = 0 \Leftrightarrow m(m^2+12m+36) = 0 \Leftrightarrow m(m+6)^2 = 0.$

$y = c_1 + c_2 e^{-6x} + c_3 x e^{-6x}.$

$y' = -6c_2 e^{-6x} - 6c_3 x e^{-6x} + c_3 e^{-6x}.$

$y'' = 36 c_2 e^{-6x} - 6c_3 e^{-6x} - 6c_3 e^{-6x} + 36 c_3 x e^{-6x}.$

$\frac{36}{2}$
72

$y(0)=0 \Rightarrow c_1 + c_2 = 0.$
$y'(0)=1 \Rightarrow -6c_2 + c_3 = 1.$
$y''(0)=-7 \Rightarrow 36c_2 - 6c_3 - 6c_3 = -7$

$\Rightarrow c_3 = 1 + 6c_2$ & $c_2 = \frac{-7+12c_3}{36}$

$= \frac{-7+12+72c_2}{36} = \frac{5}{36} + 2c_2 \Rightarrow$

$c_2 = -\frac{5}{36}$ & $c_3 = 1 - \frac{30}{36} = \frac{6}{36} = \frac{1}{6}.$

$c_1 = -c_2 \Rightarrow c_1 = \frac{5}{36}.$

$\therefore y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{x}{6}e^{-6x}.$

<u>#49, 55</u>: Find a homog. linear DE with constant coef., whose general solution is given.

49. $y = c_1 e^{x} + c_2 e^{5x} \Leftrightarrow$ aux. eq^{n} $(m-1)(m-5) = m^2 - 6m + 5$
$\Leftrightarrow y'' - 6y' + 5y = 0.$

55. $y = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$.

Here $\alpha = -1$ & $\beta = 1$, so roots $-1 \pm i$.

Aux. eq^n: $(m+1-i)(m+1+i) = m^2 + m + \cancel{im} + m + 1 \cancel{+i} - \cancel{im}$
$\cancel{-i} + 1$

$= m^2 + 2m + 2 \Leftrightarrow y'' + 2y' + 2y = 0.$