Ch. 2: First-Order DE's:

2.2: Separable Eq'n's:

#1, 5, 7, 25, ~~29~~, 31, 37, 49

1, 5, 7: Solve DE by separation of variables.

1. $\frac{dy}{dx} = \sin(5x)$

$\int dy = \int \sin(5x)\,dx$

$y = -\frac{1}{5}\cos(5x) + c.$

5. $x\frac{dy}{dx} = 4y$

$\int \frac{1}{4} y^{-1}\,dy = \int \frac{1}{x}\,dx$

$\frac{1}{4}\ln|y| = \ln|x| + c$

$\ln|y| = 4\ln|x| + c$

$y = e^{4\ln|x|+c}$

$y = e^{\ln(x^4)+c}$

$y = cx^4.$

7. $\frac{dy}{dx} = e^{3x+2y}$

$\frac{dy}{dx} = e^{3x}e^{2y}$

$\int e^{-2y}\,dy = \int e^{3x}\,dx$

$-\frac{1}{2}e^{-2y} = \frac{1}{3}e^{3x} + c$

$e^{-2y} = -\frac{2}{3}e^{3x} + c$

$-2y = \ln(-\frac{2}{3}e^{3x} + c)$

$y = -\frac{1}{2}\ln(-\frac{2}{3}e^{3x} + c)$

25. Find an explicit solution for $x^2\frac{dy}{dx} = y - xy$, $y(-1) = -1$.

$x^2\frac{dy}{dx} = y(1-x)$

$y^{-1}\,dy = x^{-2}(1-x)\,dx$

$\int y^{-1}\,dy = \int (x^{-2} - x^{-1})\,dx$

$\ln|y| = -x^{-1} - \ln|x| + c$

$\ln|x| + \ln|y| = -\frac{1}{x} + c$

$\ln|xy| = -\frac{1}{x} + c$

$xy = c\,e^{-x^{-1}}$

$y = c\,\frac{e^{-x^{-1}}}{x}.$

$-1 = c\frac{e^{-(\frac{1}{-1})}}{-1} \Leftrightarrow 1 = ce^{1} \Leftrightarrow c = e^{-1}.$ $\therefore$ $y = \frac{e^{-1-1/x}}{x}.$

31. Find an explicit solution of IVP & determine exact interval I of def^{n}. $\frac{dy}{dx} = \frac{2x+1}{2y}$, $y(-2) = -1$.

$$\int 2y\,dy = \int (2x+1)\,dx$$

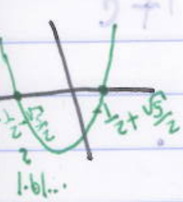
$$y^2 = x^2 + x + C$$

$$y = \pm\sqrt{x^2+x+c}.$$

$y(-2) = -1 \Leftrightarrow -1 = -\sqrt{4+(-2)+c}$

$\Leftrightarrow 1 = 2 + c$

$\Leftrightarrow c = -1.$



$\therefore \boxed{y = -\sqrt{x^2+x-1}}$.

Looking for interval I s.t. y is defined on I & s.t. y' is cont. on I.

y is defined when $x^2 + x - 1 \geq 0$.

$x = \frac{-1 \pm \sqrt{1-4(-1)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$

We want $x=-2$ to be in our interval I, so let's choose $\boxed{I = (-\infty, -\frac{1}{2} - \frac{\sqrt{5}}{2})}$.

$y' = \frac{-1}{\sqrt{x^2+x-1}} \cdot (2x+1)$ is cont. on I. ✓

49. Find an explicit solution of the IVP $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{y}$, $y(1)=4$.

$$\int y\,dy = \int e^{\sqrt{x}}\,dx$$

$w = \sqrt{x}$

$dw = \frac{1}{2\sqrt{x}}dx$ $\quad \frac{1}{2}y^2 = \int e^{w}(2w)\,dw$

$dw = \frac{1}{2w}dx$ $\quad \frac{1}{2}y^2 = 2\int we^{w}\,dw$

$2wdu = dx$ $\quad \frac{1}{2}y^2 = 2\left[we^{w} - \int e^{w}dw\right]$

$u = w \quad v = e^{w}$ $\quad \frac{1}{2}y^2 = 2we^{w} - 2e^{w} + C$

$du = dw \quad dv = e^{w}$

$$\frac{1}{2}y^2 = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$$y = \pm\sqrt{4\sqrt{x}\,e^{\sqrt{x}} - 4e^{\sqrt{x}} + C}.$$

$$y(1) = 4 \Rightarrow 4 = \sqrt{4e - 4e + C} \Rightarrow 16 = C.$$

$$\therefore\ y = \sqrt{4\sqrt{x}\,e^{\sqrt{x}} - 4e^{\sqrt{x}} + 16} = 2\sqrt{\sqrt{x}\,e^{\sqrt{x}} - e^{\sqrt{x}} + 4}.$$