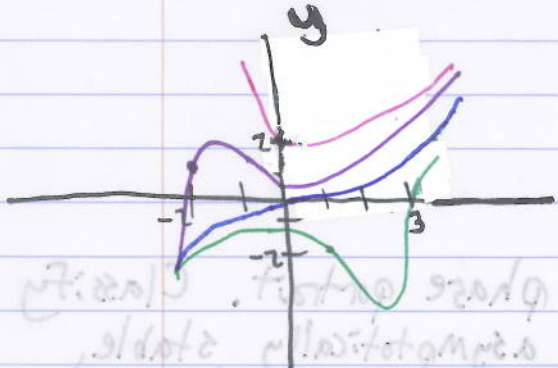
2.1: Solution Curves Without a Solution

#1, 15, 21, 23, 25, 29

1. Sketch by hand an approx. solution curve that passes through each of the indicated points.



ⓐ $y(-2)=1$ ⓑ $y(3)=0$

ⓒ $y(0)=2$ ⓓ $y(0)=0$

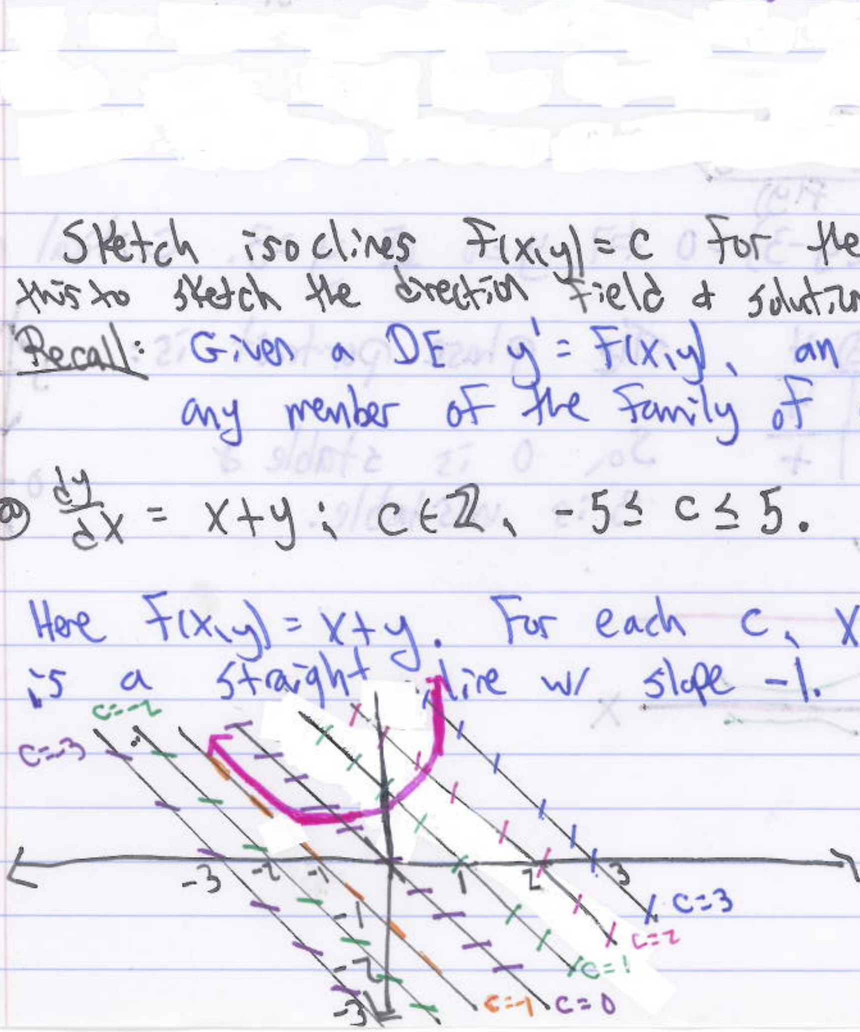
* see textbook for what the given direction field looks like *

15. Sketch isoclines $F(x,y)=c$ for the given DE. Use this to sketch the direction field & solution curve for IVP $y(0)=1$.

Recall: Given a DE $y'=F(x,y)$, an isocline is any member of the family of curves $F(x,y)=c$, $c\in\mathbb{R}$.

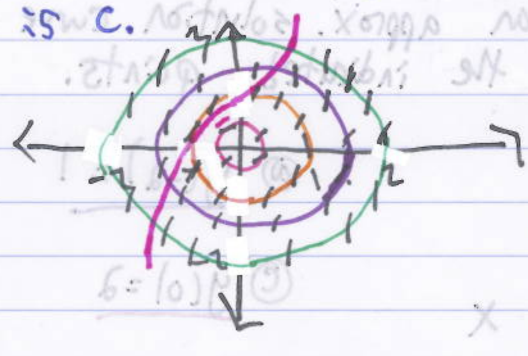
ⓐ $\frac{dy}{dx}=x+y$; $c\in\mathbb{Z}$, $-5\le c\le 5$.

Here $F(x,y)=x+y$. For each c, $x+y=c \iff y=-x+c$ is a straight line w/ slope -1.



(b) $\frac{dy}{dx} = x^2 + y^2$; $c = 1/4$, $c = 1$, $c = 9/4$, $c = 4$.

$x^2+y^2 = c$ is a circle of radius $\sqrt{c}$.
Slope of direction field along each isocline corr. to c is c.



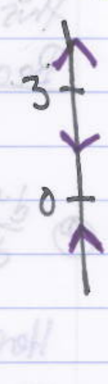
21, 23, 25: Find critical pts & phase portrait. Classify each critical pt as asymptotically stable, unstable, or semi-stable. Sketch typical solution curves.

21. $\frac{dy}{dx} = \underbrace{y^2 - 3y}_{f(y)}$

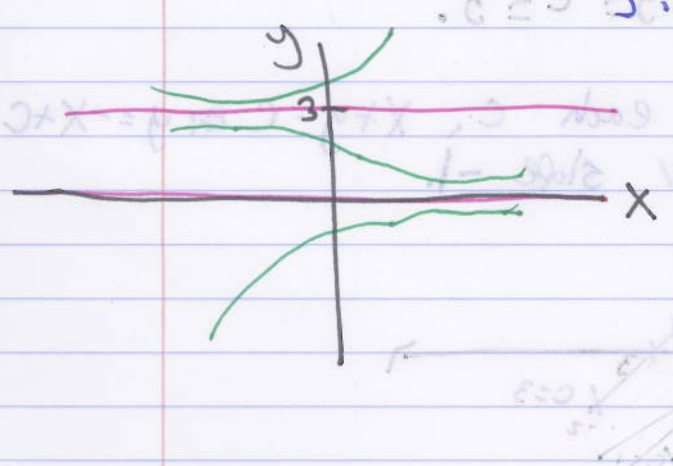
$y(y-3) = 0 \iff y = 0$ or $y = 3$. Critical points.

y	-1	0	1	3	4
$f(y)$	4		-2		4
sign	$+$		$-$		$+$

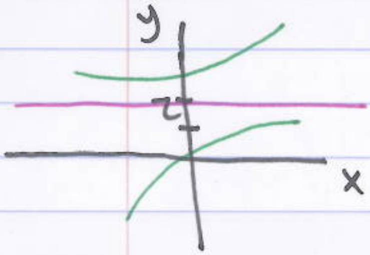
The phase portrait is:



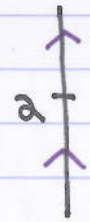
So, 0 is stable & 3 is unstable.



23. $\frac{dy}{dx} = (y-2)^4$. One critical pt $y=2$.



y	0	2	3
$F(y)$	16		1
sign	+		+

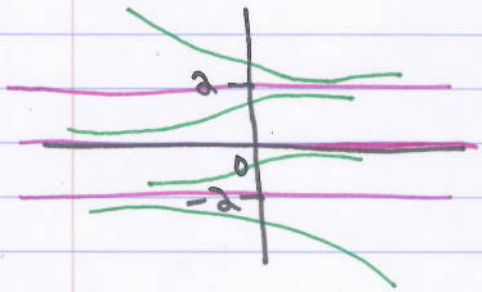


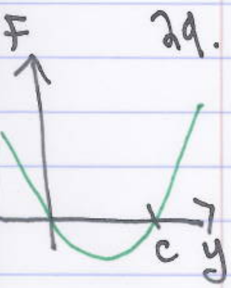
Semistable

25. $\frac{dy}{dx} = y^2(4-y^2)$. $y^2(4-y^2) = 0 \iff y=0$ or $y=2$ or $y=-2$.

-3	-2	-1	0	1	2	3
−		+		+		−

2 stable, 0 semi-stable, -2 unstable.



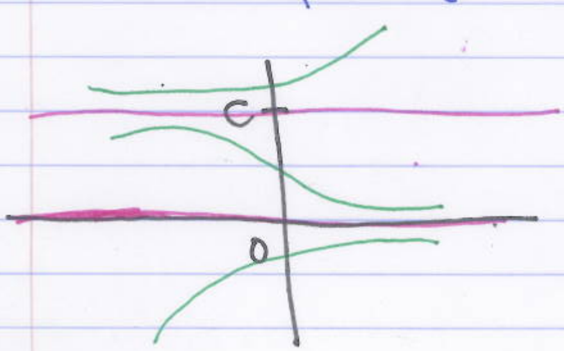


29. Consider the autonomous DE $\frac{dy}{dx} = F(y)$, where the graph of F is given. Locate the critical pts of the DE, sketch a phase portrait & sketch typical solution curves.

Critical pts are 0 & c.

-1	0	1	c	c+1
+		−		+

looking to see if F is pos. or neg.



0 stable & c unstable.