1.2: IVP's

5, 7, 15, 19, 25, 27, 49

5. $y = \frac{1}{x^2+C}$ is a 1-parameter family of solutions of $y' + 2xy^2 = 0$. Find a solution of IVP $y(0)=1$ & give largest interval over which solution is defined.

$y(0)=1 \Rightarrow 1 = \frac{1}{0^2+C} \Rightarrow 1=\frac{1}{C} \Rightarrow C=1$. $\therefore$ $y = \frac{1}{x^2+1}$.

$y' = -(x^2+1)^{-2} \cdot 2x = \frac{-2x}{(x^2+1)^2}$. Need largest interval y defined & y' cont..

y defined everywhere, since $x^2+1 > 0$. y' defined everywhere too & quotient of polynomials $\Rightarrow$ cont..
$\therefore$ $y = \frac{1}{x^2+1}$ solution on $(-\infty, \infty)$.

7. $x = c_1 \cos t + c_2 \sin t$ 2-par. family of solutions of 2nd-order DE $x'' + x = 0$. Find a solution of IVP for $x(0) = -1$, $x'(0) = 8$.

$x(0) = -1 \Rightarrow -1 = c_1 \cos(0) + c_2 \sin(0) \Rightarrow -1 = c_1$.

$x' = -c_1 \sin t + c_2 \cos t$. $x'(0) = 8 \Rightarrow 8 = c_2$.

$\therefore$ $x = -\cos t + 8 \sin t$ solution (on $(-\infty, \infty)$).

15. Determine by inspection 2 solutions of IVP $y' = 3y^{2/3}$, $y(0) = 0$.

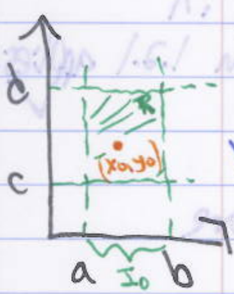
The constant function $y = 0$ is s.t. $y(0) = 0$ & $y' = 0$, so $y' = 3y^{2/3} \iff 0 = 0$. ✓

Try: $y = x^3$. $y(0) = 0$. ✓ $y' = \underbrace{3x^2}$. $3y^{2/3} = \underbrace{3x^2}$. same ✓

$\therefore y = 0$ & $y = x^3$ are 2 solutions of the IVP.

19. Determine a region of the xy-plane for which the DE $x\frac{dy}{dx} = y$ has a unique solution whose graph passes through a pt (x_0, y_0) in the region.

Recall: Theorem 1.2.1: Existence of a Unique Solution:



Let R be a rectangular region in the xy-plane defined by $a \le x \le b$, $c \le y \le d$ that contains the pt (x_0, y_0) in its interior. If $f(x, y)$ & $\frac{\partial f}{\partial y}$ are cont. on R, then $\exists$ an interval I_0: $(x_0 - h, x_0 + h)$, $h > 0$, contained in $[a, b]$ & a unique function $y(x)$ defined on I_0 that is a solution to the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

$x\frac{dy}{dx} = y \iff \frac{dy}{dx} = \frac{y}{x}$. So, here

$f(x, y) = \frac{y}{x}$. $\frac{\partial f}{\partial y} = x^{-1}$.

f is cont. on $(0, \infty)$ & $(-\infty, 0)$. Same w/ $\frac{\partial f}{\partial y}$.

i.e. there is some I_0 in those regions.

$\therefore$ The regions $(0, \infty)$ & $(-\infty, 0)$ would both guarantee a unique solution. We could choose either as an answer.

$\boxed{25-27}$: Determine whether Theorem 1.2.1 guarantees that the DE $y' = \sqrt{y^2-9}$ possesses a unique solution through the given pt.

25. $\underline{(1,4)}$: $y' = \sqrt{y^2-9} \Rightarrow F(x,y) = \sqrt{y^2-9}$.

$\frac{\partial F}{\partial y} = \frac{1}{2}(y^2-9)^{-1/2} \cdot 2y = \frac{y}{\sqrt{y^2-9}}$.

F defined when $y^2 - 9 \geq 0 \Leftrightarrow y^2 \geq 9 \Leftrightarrow |y| \geq 3$
$\Leftrightarrow y \geq 3$ or $y \leq -3$. $\therefore F$ cont. on $[3,\infty)$ & $(-\infty,-3]$.

For $\frac{\partial F}{\partial y}$ we also need $y^2 - 9 \neq 0 \Leftrightarrow y \neq 3$ & $y \neq -3$.

i.e. F & $\frac{\partial F}{\partial y}$ are both cont. on $(-\infty,-3)$ & $(3,\infty)$.

$\therefore$ The only intervals where the Theorem applies are $(3,\infty)$ & $(-\infty,-3)$, and $y=4$ is in $(3,\infty)$,
so Yes, we could choose an interval R in that contains $(1,4)$ for which Theorem 1.2.1 applies. ✓

27. $\underline{(2,-3)}$: By our computation in #25, F & $\frac{\partial F}{\partial y}$ are only cont. for $\{y \mid y < -3 \text{ or } y > 3\}$.

Since $y=-3$ is not in this range $\Rightarrow$ we can't use Theorem 1.2.1 to guarantee a unique solution. $\therefore$ No
(i.e. no guarantee of a unique solution).