

Math 2C03: Quiz #6

WEDNESDAY, JULY 29TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: *marking Scheme* Student ID: _____

Please answer each question fully, providing all work and reasoning.

Questions:

1. (12pts) The differential equation $(1+x^2)y'' - y' + y = 0$ has power series solution $y = \sum_{n=0}^{\infty} c_n x^n$, where the recursive formula for the coefficients c_n is:

$$c_{n+2} = \frac{(n+1)c_{n+1} - (n^2 - n + 1)c_n}{(n+2)(n+1)}, n \geq 2,$$

$$2c_2 - c_1 + c_0 = 0, 6c_3 - 2c_2 + c_1 = 0.$$

Using this, write the first 4 terms for a *general solution* to this DE. How do you know this is a general solution?

This needs
to be in terms
of c_0 & c_1
alone, since a
general solution to
this DE
has 2 arbitrary
constants.
The first

General
solution b/c:

$$2c_2 = c_1 - c_0 \Rightarrow c_2 = \frac{1}{2}c_1 - \frac{1}{2}c_0.$$

$$6c_3 = 2c_2 - c_1 \Rightarrow c_3 = \frac{1}{3}c_2 - \frac{1}{6}c_1 = \frac{1}{3}\left[\frac{1}{2}c_1 - \frac{1}{2}c_0\right] - \frac{1}{6}c_1 = -\frac{1}{6}c_0.$$

4 terms of the general solution are: [6pts]

$$y = c_0 + c_1 x + \left(-\frac{1}{2}c_0 + \frac{1}{2}c_1\right)x^2 - \frac{1}{6}c_0 x^3 + \dots$$

Since the eqⁿ is order 2, we know the space of solutions has dimension 2.

Consider $y_1 = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \dots$ [setting $c_0=1, c_1=0$]

+ $y_2 = x + \frac{1}{2}x^2 + \dots$ [setting $c_0=0, c_1=1$].

1

y_1 & y_2 are linearly independent [y_1 has a constant term & y_2 doesn't \Rightarrow don't differ by a scalar multiple] & both are solutions since y is a solution for any choice of c_0 & c_1 . back of pg.

$\therefore \{y_1, y_2\}$ form a fundamental set of solutions for this DE, & $y = c_0 y_1 + c_1 y_2 \Rightarrow y$ is the general solution to this DE. [6pts]

[They can pick any choices of c_0 & c_1 to make the linear independence argument, so long as y_1 & y_2 are linearly independent].

$$P_3^2 - P_2 P_4 = P_3^2 - P_2^2 = 30 \quad P_2 P_3 - P_1 P_4 = P_2^2 - P_1^2 = 30$$

$$P_2^2 - [P_3^2 - P_2 P_4] = P_2^2 - P_2^2 = 0 \quad P_1 P_3 - P_0 P_4 = P_1^2 - P_0^2 = 0$$

$$[279d] \quad \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

To solve this work with x, y, z, w in "row" order

... & normal and transpose

$$[0=0, 1=0 \text{ pattern}] \quad \dots + y z^2 - x z^2 - 1 = 0 \quad \text{Transcendental}$$

$$[0=0, 0=0 \text{ pattern}] \quad \dots + y z^2 + x = 0$$

sketch below and it is probably easiest to use x, y, z, w in "row" order to get a good understanding of what is going on. It is possible to do this with x, y, z, w in "column" order, but it is much more difficult.