Math 2C03: Quiz #6

WEDNESDAY, JULY 29TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: *marking scheme* Student ID: ____________

Please answer each question fully, providing all work and reasoning.

Questions:

1. (12*pts*) The differential equation $(1+x^2)y'' - y' + y = 0$ has power series solution $y = \sum_{n=0}^{\infty} c_n x^n$, where the recursive formula for the coefficients c_n is:

$c_{n+2} = \dfrac{(n+1)c_{n+1} - (n^2 - n + 1)c_n}{(n+2)(n+1)}, n \geq 2,$

$2c_2 - c_1 + c_0 = 0, 6c_3 - 2c_2 + c_1 = 0.$

Using this, write the first 4 terms for a *general solution* to this DE. How do you know this is a general solution?

This needs to be in terms of c_0 & c_1 alone, since a general solution to this DE has 2 arbitrary constants.

$2c_2 = c_1 - c_0 \Rightarrow c_2 = \frac{1}{2}c_1 - \frac{1}{2}c_0.$

$6c_3 = 2c_2 - c_1 \Rightarrow c_3 = \frac{1}{3}c_2 - \frac{1}{6}c_1 = \frac{1}{3}\left[\frac{1}{2}c_1 - \frac{1}{2}c_0\right] - \frac{1}{6}c_1 = -\frac{1}{6}c_0.$

The first 4 terms of the general solution are: [6pts]

$y = c_0 + c_1 x + \left(-\frac{1}{2}c_0 + \frac{1}{2}c_1\right)x^2 - \frac{1}{6}c_0 x^3 + \cdots.$

General Solution b/c:

Since the eq'n is order 2, we know the space of solutions has dimension 2.

Consider $y_1 = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \ldots$ [setting $c_0 = 1, c_1 = 0$]

& $y_2 = x + \frac{1}{2}x^2 + \ldots$ [setting $c_0 = 0, c_1 = 1$].

y_1 & y_2 are linearly independent [y_1 has a constant term & y_2 doesn't $\Rightarrow$ don't differ by a scalar multiple] & both are solutions since y is a solution for any choice of c_0 & c_1.

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$\therefore \{y_1, y_2\}$ Form a Fundamental set of solutions for this DE, & $y = c_0 y_1 + c_1 y_2 \Rightarrow y$ is the general solution to this DE. [6pts]

[They can pick any choices of c_0 & c_1 to make the linear independence argument, so long as y_1 & y_2 are linearly independent].