Math 2C03: Quiz #5

MONDAY, JULY 27TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: *Marking scheme* Student ID: ______

Please answer each question fully, providing all work and reasoning.

Questions:

[4pts] 1. Compute $\mathscr{L}\{te^{-3t}\cos(3t)\}$ using the Laplace transform table provided.

$$\mathscr{L}\{te^{-3t}\cos(3t)\} = \frac{-d}{ds}\mathscr{L}\{e^{-3t}\cos(3t)\}$$

[2pts]

$$= -\frac{d}{ds}\mathscr{L}\{\cos(3t)\}\Big|_{s\to s+3} = -\frac{d}{ds}\frac{(s+3)}{(s+3)^2+9}$$

[2pts]

$$= \frac{-[(s+3)^2+9] + (s+3)[2(s+3)]}{[(s+3)^2+9]^2} = \frac{(s+3)^2-9}{[(s+3)^2+9]^2}.$$

[8pts] 2. Verify that the power series

$$y=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n}(n!)^2}x^{2n}$$

is a solution to the differential equation $xy''+y'+xy=0$.

Hint: You'll want to make a substitution $k=n$ and $k=n+1$.

[2pts]

$$y=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n}(n!)^2}x^{2n},\quad y'=\sum_{n=1}^{\infty}\frac{(-1)^n 2n}{2^{2n}(n!)^2}x^{2n-1},\quad y''=\sum_{n=1}^{\infty}\frac{(-1)^n 2n(2n-1)}{2^{2n}(n!)^2}x^{2n-2}$$

$$xy''+y'+xy=\underbrace{\sum_{n=1}^{\infty}\frac{(-1)^n 2n(2n-1)}{2^{2n}(n!)^2}x^{2n-1}}_{k=n}+\underbrace{\sum_{n=1}^{\infty}\frac{(-1)^n 2n}{2^{2n}(n!)^2}x^{2n-1}}_{k=n}$$

$$+\underbrace{\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n}(n!)^2}x^{2n+1}}_{k=n+1}$$

[2pts]

$$=\sum_{k=1}^{\infty}\frac{(-1)^k 2k(2k-1)}{2^{2k}(k!)^2}x^{2k-1}+\sum_{k=1}^{\infty}\frac{(-1)^k 2k}{2^{2k}(k!)^2}x^{2k-1}+\sum_{k=1}^{\infty}\frac{(-1)^{k-1}x^{2k-1}}{2^{2(k-1)}[(k-1)!]^2}$$

$$=\sum_{k=1}^{\infty}\frac{(-1)^k[4k^2]}{2^{2k}(k!)^2}x^{2k-1}-\sum_{k=1}^{\infty}\frac{(-1)^k k^2}{2^{2k}\cdot 2^{-2}[k!]^2}x^{2k-1}$$ [2pts]

$$=\sum_{k=1}^{\infty}\frac{(-1)^k[4k^2]}{2^{2k}(k!)^2}x^{2k-1}-\sum_{k=1}^{\infty}\frac{(-1)^k 4k^2}{2^{2k}(k!)^2}x^{2k-1}=0.\ \checkmark$$ [2pts]

$\therefore$ y is a solution to this DE.