Math 2C03: Quiz #4

MONDAY, JULY 20TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: * Marking Scheme * Student ID: _____

Please answer each question fully, providing all work and reasoning.

Questions:

1. Using the Laplace transform table provided, compute the following:
   (a) (3*pts*) $\mathscr{L}\{t^2 - e^{-9t} + 5\}$.
   (b) (3*pts*) $\mathscr{L}^{-1}\left\{\frac{1}{4s^2+1}\right\}$.

[a] $\mathscr{L}\{t^2 - e^{-9t} + 5\} = \mathscr{L}\{t^2\} - \mathscr{L}\{e^{-9t}\} + 5\mathscr{L}\{1\}$

$= \frac{2}{s^3} - \frac{1}{s+9} + \frac{5}{s}$.

[b] $\mathscr{L}^{-1}\left\{\frac{1}{4s^2+1}\right\} = \frac{1}{4}\mathscr{L}^{-1}\left\{\frac{1}{s^2+\frac{1}{4}}\right\} = \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{\frac{1}{2}}{s^2+(\frac{1}{2})^2}\right\}$

$= \frac{1}{2}\sin\left(\frac{t}{2}\right)$.

2. (6*pts*) Solve The IVP $y'' - y' - 6y = 0, y(0) = 2, y'(0) = -1$.

(Hint: $\frac{2s-3}{(s-3)(s+2)} = \frac{\frac{3}{5}}{s-3} + \frac{\frac{7}{5}}{s+2}$. This can be computed using partial fractions, but you're not required to do so here.)

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 6\mathcal{L}\{y\} = 0$$

$$[s^2 Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-1}] - [sY(s) - \underbrace{y(0)}_{2}] - 6Y(s) = 0$$

$$Y(s)[s^2 - s - 6] - 2s + 3 = 0$$

$$Y(s) = \frac{2s-3}{(s-3)(s+2)} = \frac{3/5}{s-3} + \frac{7/5}{s+2}$$

$$\Rightarrow y(t) = \frac{3}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \frac{7}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$\Rightarrow y(t) = \frac{3}{5}e^{3t} + \frac{7}{5}e^{-2t}.$$