Math 2C03: Quiz #3

MONDAY, JULY 13TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: *Marking scheme* Student ID: ______________

Please answer each question fully, providing all reasoning

Questions:

1. (a) (2pts) What is a *fundamental set of solutions*?
   (b) (2pts) Suppose f_1, f_2, and f_3 are solutions to a second-order linear homogeneous differential equation. Is $\{f_1, f_2, f_3\}$ a fundamental set of solutions? Why or why not?

[2pts] a A Fundamental set of solutions on an interval I is any set $\{y_1, \ldots, y_n\}$ of n linearly independent solutions of the homogeneous linear n^{th}-order DE $\underbrace{a_n(x)y^{(n)} + \ldots + a_0(x)y = 0}_{L(y)=0}$ on an interval I.

[either of these would be fine].

i.e.) It's a basis for the space of solutions of $L(y) = 0$, on I.

[2pts] $\boxed{b}$ The dimension of the solution space of a 2^{nd}-order linear homog. DE is 2. Therefore, any basis (Fundamental set of solutions) has exactly 2 elements. $\therefore \{f_1, f_2, f_3\}$ is not a Fundamental set of solutions, b/c it has 3 elements.

2. (a) (4pts) Give an example of two functions which are linearly independent. Explain why they're linearly independent.

   (b) (4pts) Give an example of two functions which are linearly dependent. Explain why they're linearly dependent.

$\boxed{a}$ x & x^2 are linearly independent on $(-\infty,\infty)$. [2pts]

[2pts] This is b/c, if $c_1 x + c_2 x^2 = 0 \ \forall x \in (-\infty,\infty)$ $\Rightarrow c_1 = 0$ & $c_2 = 0$.

$\boxed{b}$ x & $2x$ are linearly dependent on $(-\infty,\infty)$. [2pts]

[2pts] This is b/c, we could choose $c_1 = -2$ & $c_2 = 1$. Then $c_1 x + c_2(2x) = -2x + 2x = 0$ on $(-\infty,\infty)$. $\therefore$ linearly dependent.

* could choose any pair of functions that work here. Also in $\boxed{b}$, may argue that the 2 functions aren't a constant multiple of each other. *

<u>Note</u>: The Wronskian can't be used here for checking linear independence, unless you also show that the 2 functions satisfy a homog. linear DE, $L(y) = 0$, b/c Theorem 4.1.3 only works when y_1 & y_2 are solutions to $L(y) = 0$.