Math 2C03: Quiz #2

MONDAY, JULY 6TH, 7PM (FIRST 10 MINUTES OF CLASS)
McMaster University

Name: *marking scheme* Student ID: _____

Please answer each question fully, providing all reasoning

Questions:

1. In class last Monday we discussed three types of differential equations: homogeneous, Bernoulli's equation, and DE's of the form $y' = f(Ax + By + C)$, for $A, B, C \in \mathbb{R}$, $B \neq 0$. These equations can all be solved by making an appropriate substitution which transforms them into a separable or linear equation.

   (a) (3pts) Define each type of equation.

   (b) (3pts) In each case, what substitution should be made to solve it?

   (c) (3pts) For each, which type of equation does the differential equation become after making that substitution?

   (d) (3pts) Give an example of each type of equation.

(a) (1pt) A 1st-order DE $M(x,y)dx + N(x,y)dy = 0$ is homogeneous if M & N are homogeneous functions of the same degree. i.e. if $M(tx,ty) = t^\alpha M(x,y)$ & $N(tx,ty) = t^\alpha N(x,y)$ for some $\alpha \in \mathbb{R}$.

(1pt) Bernoulli's eq'n is a DE of the form $y' + P(x)y = f(x)y^n$.

(1pt) $y' = F(Ax+By+C)$ is a 1st-order DE $y' = g(x,y)$, where $g(x,y)$ can be written as a function of $u = Ax+By+C$.

[For this one, the name $y' = F(Ax+By+C)$ is kind of self-explanatory... so even if they say its a DE of the form $y' = F(Ax+By+C)$, that would be fine].

(b) (1pt) Make the substitution $y = ux$ or $x = vy$.

(1pt) $u = y^{1-n}$

(1pt) $u = Ax + By + C$

(c) (4pt)  ⇝ Separable

(4pt)  ⇝ linear

(4pt)  ⇝ Separable

(d) (4pt) $\frac{dy}{dx} = \frac{x^2-y^2}{3xy} \iff 3xy\,dy + (-x^2+y^2)\,dx$ homog. of degree 2.

(4pt)  $y' - 5y = -\frac{5}{2}xy^3$ Bernoulli's eq^{n}, with $n=3$.

(4pt)  $y' = (-2x+y)^2 - 7$ has the form $y' = F(Ax+By+c)$,
where $A=-2$, $B=1$, $c=0$.

* any examples that work are Fine here.*