# Math 2C03: Quiz #1

MONDAY, JUNE 29TH, 7PM (FIRST 10 MINUTES OF CLASS)
*McMaster University*

Name: * Marking Scheme * Student ID: ____________

Please answer each question fully, providing all reasoning.

## Questions:

1. (a) (2*pts*) What is an $n^{th}$-order *linear* differential equation?
   (b) (2*pts*)Give an example of a first-order *linear* and a *nonlinear* DE.
   (c) (2*pts*) Describe how you would find an explicit solution for a first-order linear DE.

[2pts] (a) An $n^{th}$-order DE $F(x, y, y', \ldots, y^{(n)}) = 0$ is <u>linear</u> if $F$ is linear in the variables $y, y', \ldots, y^{(n)}$.

stating either of these is correct. (2pts)

<u>i.e.</u>, $F(x, y, y', \ldots, y^{(n)}) = a_n(x) y^{(n)} + a_{n-1}(x) y^{(n-1)} + \ldots + a_1(x) y' + a_0(x) y = g(x)$.

$\boxed{b}$ An example of a $1^{st}$-order linear DE would be $x + 2 + xy = 0$. [1pt] A $1^{st}$-order nonlinear DE would be $yy' + x = 0$. [1pt]

nonlinear term

* Any examples would be fine here. $1^{st}$-order linears have form $a_1(x) y' + a_0(x) y = g(x)$.

$1^{st}$-order nonlinear is a $1^{st}$-order DE ($y'$ highest derivative) & fails to be linear. *

[2pts] $\boxed{c}$ Put linear DE in form $y' + P(x) y = F(x)$.

An explicit solution is given by $y = e^{-\int P(x)dx} \left[ \int e^{\int P(x)dx} F(x)\, dx \right]$ & is defined on an interval where $y$ is $C^1$ (<u>i.e.</u>, where $P(x)$ & $F(x)$ are continuous).



<u>or</u> you might say: multiply both sides of $y' + P(x) y = F(x)$ by the integrating factor $e^{\int P(x)dx}$ & then integrate both sides of $\frac{d}{dx}\left[ e^{\int P(x)dx} y \right] = e^{\int P(x)dx} F(x)$ & solve for $y$.

2. (*6pts*) Give an example of a first-order *exact* differential equation. Give an example of a first-order differential equation that is NOT exact. Explain your reasoning.

[3pts] An example of a 1st-order exact eq'n would be $2y\,dx + 2x\,dy = 0$, since the differential of the function $F(x,y) = 2xy$ is $2y\,dx + 2x\,dy$.

[or could justify it by Criterion for an Exact DE: $M$ & $N$ are both $C^1$, so $M\,dx + N\,dy$ exact $\iff \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
Here $\frac{\partial M}{\partial y} = 2 = \frac{\partial N}{\partial x} \Rightarrow$ exact].

[2 pts for giving an exact DE & 1pt for justifying that it's exact].

[3pts] An example of a 1st-order non-exact DE would be $\underbrace{2y^2}_{M}\,dx + \underbrace{2x}_{N}\,dy = 0$. $M$ & $N$ are both $C^1$ here,

So $\frac{\partial M}{\partial y} = 4y \neq \frac{\partial N}{\partial x} = 2 \Rightarrow$ this DE is <u>not</u> exact.

[2pts for giving a correct example & 1pt for justifying that it's <u>not</u> exact].