Math 1B03/1ZC3 - Tutorial 9

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Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .

Examples:

- **1.** Find a unit vector that has the same direction as $(-4, -3)$.
- **Recall:** a vector of norm 1 is called a unit vector. i.e. if $||u|| = 1$, then u is a unit vector.

Examples:

- **2.** Let $u = (0, 2, 2, 1)$ and $v = (1, 1, 1, 1)$. Verify that the Cauchy-Schwartz inequality holds.
- **Recall: Cauchy-Schwartz Inequality:** $|u \cdot v| \leq ||u||||v||$.

Examples:

- **3.** Suppose $||u|| = 2, ||v|| = 1$, and $u \cdot v = 1$. What is the angle in radians between u and v?

- **Recall:** $\cos\theta = \frac{u \cdot v}{||u||||v||}$.

Examples:

- **4.** Let $u = (1, 0, 1)$ and $v = (0, 1, 1)$.
- **a)** Find two unit vectors orthogonal to both u and v.
- **Recall:** Two vectors u and v are **orthogonal** if $u \cdot v = 0$.
- **b)** Do u, v, and one of the unit vector you found form an orthogonal set?
- **Recall:** A nonempty set of vectors in $\mathbb{R}^n$ is called an orthogonal set if all pairs of distinct vectors in the set are orthogonal.

Examples:

- **5.** What does the equation $-2(x+1)+(y-3)-(z+2)=0$ represent geometrically?

- **Recall:** The **point normal equation** of a plane is: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$, where $P_0(x_0,y_0,z_0)$ is a specific point on the plane, $P=(x,y,z)$ is an arbitrary point on the plane, and $n=(a,b,c)$ is the normal vector to the plane.

Examples:

- **6.** Let $u = (6,2)$ and $a = (3,-9)$.
- **a)** Find the vector component of u along a.
- **Recall:** If u and a are vectors in $\mathbb{R}^n$ such that $a \neq 0$, then we can write $u = w_1 + w_2$, where $w_1 = proj_a u = \frac{u \cdot a}{||a||^2} a$ (vector component of u along a; a.k.a. orthogonal projection of u along a),
  and $w_2 = u - w_1 = u - proj_a u$ (component of u orthogonal to a).
- **b)** Find the vector component of u orthogonal to a.

Examples:

- **7.** Find the distance between the point $(3,1,-2)$ and the plane $x+2y-2z=4$.
- **Recall:)** In $\mathbb{R}^3$, the distance between a point $P_0(x_0,y_0,z_0)$ and a plane $ax+by+cz+d=0$ is: $\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$.

Examples:

- **8.** Consider two points $P(2,3,-2)$ and $Q(7,-4,1)$. Find the point on the line segment containing P and Q that is $\frac{3}{4}$ of the way from P to Q.
- **Recall:** The vector with initial point $P_1(x_1,y_1,z_1)$ and terminal point $P_2(x_2,y_2,z_2)$ is given by the formula: $\overrightarrow{P_1P_2} = (x_2-x_1, y_2-y_1, z_2-z_1)$.

Examples:

- **9. a)** What is the vector equation of the line $4y + 3x = 40$?

- **Recall:** The vector equation of a line ℓ through the point x_0 that is parallel to v is $\ell = x_0 + tv$. i.e. v gives direction and x_0 gives position.
- **b)** What are the parametric equations of this line?
- **c)** Which line passes through $(1,2)$ and is parallel to ℓ?

- **Recall:** Two lines are parallel if their direction vectors are multiples of each other.
- **d)** Find a line that is perpendicular to ℓ.

- **Recall:** Two lines are perpendicular if their dot product is zero.

Examples:

- **10.** Find a vector equation of the plane in $\mathbb{R}^4$ passing through the point $(2,-1,7,3)$ and parallel to both $(1,0,2,1)$ and $(3,2,4,5)$.
- **Recall:** The equation of a plane passing through a point x_0 and parallel to v_1 and v_2, where v_1 and v_2 are not collinear, is $x = x_0 + v_1 t + v_2 s$.

Examples:

- **11.** Find the area of the triangle with vertices $P = (1,1,5)$, $Q = (3,4,3)$, and $R = (1,5,7)$.

- **Recall:** If u and v are vectors in 3-space, then $||u \times v||$ = area of the parallelogram determined by u and v.

- **Recall:**

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix},$$

where i, j, and k are the standard unit vectors

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$