Math 1B03/1ZC3 - Tutorial 7

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Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca .

Examples:

- **1.** Suppose the population of raccoons in the city in 2010 is 100 and the population of raccoons in the nearby forest is 300. Suppose we also know that 10% of the racoons in the forest move to the city, and 5% of the raccoons in the city move to the forest each year.
- **a)** Set up a transition matrix to describe this phenomenon.
- **Recall:** Our transition matrix takes us from time k to time $k+1$:

$$\begin{pmatrix} w_{k+1} \\ c_{k+1} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} w_k \\ c_k \end{pmatrix}.$$

- **b)** Is T a regular stochastic matrix?
- **Recall:** A square matrix A is called a **stochastic matrix** is each of its columns is a probability vector (i.e. the entries of each column sum to 1).
- **c)** Does T have a steady-state vector? If so, what is it?
- **Recall:** If P is a regular transition matrix for a Markov chain, then $\exists!$ probability vector q such that $Pq = q$ (i.e. q is an eigenvector corresponding to $\lambda = 1$ and q's entries sum to 1). This vector is called the **steady-state vector**.

- **Recall:** A stochastic matrix A is called **regular** if A, or some positive power of A, has all positive entries.
- **d)** In the long term, how will the population of raccoons in the city and woods be distributed?
- **Recall:** If q is a steady-state vector for a regular Markov chain, then for any initial probability vector x_0, $\lim_{k\to\infty} P^k x_0 = q$, where P is the transition matrix for this chain.
- **e)** How many raccoons will be in the city after 20 years?

- **Recall:** We know $x_n = P^n x_0$, where x_0 is the initial state vector, x_n is the state vector at time n, and P is the transition matrix.

Examples:

- **2.** Express $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$ as a real number.

Examples:

- **3.** Consider $z = \frac{i}{-2-2i}$.
- **a)** Express z in rectangular form.
- **b)** Express z in polar form.
- **c)** What is $\operatorname{Arg} z$?
- **Recall:** The **argument** of z is multivalued, i.e. $\arg z = \theta + 2\pi k, k \in \mathbb{Z}$.
- The **principal argument**, $\operatorname{Arg} z$, is such that $-\pi < \operatorname{Arg} z \leq \pi$.
- **d)** What is $\bar{z}$?
- **Recall:** If $z = a + bi$, then the **complex conjugate** of z is: $\bar{z} = a - bi$.

Examples:

- **4.** Express $(\sqrt{3} - i)^6$ in polar form.

Examples:

- **5.** Find the solutions to the equation $z^3 = -1$.
- **Recall:** $z^{\frac{1}{n}} = \sqrt[n]{r}[\cos(\frac{\theta}{n} + \frac{2k\pi}{n}) + i\sin(\frac{\theta}{n} + \frac{2k\pi}{n})]$, $k = 0, 1, \ldots, n-1$.

Examples:

- **6. a)** Find the square roots of $2i$.
- **b)** Express your two roots in rectangular coordinates.