

# Math 1B03/1ZC3 - Tutorial 7



Feb. 28th/ Mar. 4th, 2014

## **Tutorial Info:**

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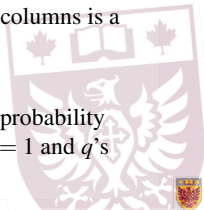
## Examples:

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- **1.** Suppose the population of raccoons in the city in 2010 is 100 and the population of raccoons in the nearby forest is 300. Suppose we also know that 10% of the racoons in the forest move to the city, and 5% of the raccoons in the city move to the forest each year.
- **a)** Set up a transition matrix to describe this phenomenon.
- **Recall:** Our transition matrix takes us from time  $k$  to time  $k + 1$ :

$$\begin{pmatrix} w_{k+1} \\ c_{k+1} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} w_k \\ c_k \end{pmatrix}.$$

- **b)** Is  $T$  a regular stochastic matrix?
- **Recall:** A square matrix  $A$  is called a **stochastic matrix** if each of its columns is a probability vector (i.e. the entries of each column sum to 1).
- **c)** Does  $T$  have a steady-state vector? If so, what is it?
- **Recall:** If  $P$  is a regular transition matrix for a Markov chain, then  $\exists!$  probability vector  $q$  such that  $Pq = q$  (i.e.  $q$  is an eigenvector corresponding to  $\lambda = 1$  and  $q$ 's entries sum to 1). This vector is called the **steady-state vector**.



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- **Recall:** A stochastic matrix  $A$  is called **regular** if  $A$ , or some positive power of  $A$ , has all positive entries.
  - **d)** In the long term, how will the population of raccoons in the city and woods be distributed?
  - **Recall:** If  $q$  is a steady-state vector for a regular Markov chain, then for any initial probability vector  $x_0$ ,  $\lim_{k \rightarrow \infty} P^k x_0 = q$ , where  $P$  is the transition matrix for this chain.
  - **e)** How many raccoons will be in the city after 20 years?
  - **Recall:** We know  $x_n = P^n x_0$ , where  $x_0$  is the initial state vector,  $x_n$  is the state vector at time  $n$ , and  $P$  is the transition matrix.



## Examples:

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- 2. Express  $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$  as a real number.



## Examples:

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- 3. Consider  $z = \frac{i}{-2-2i}$ .
- a) Express  $z$  in rectangular form.
- b) Express  $z$  in polar form.
- c) What is  $\text{Arg } z$ ?
- **Recall:** The **argument** of  $z$  is multivalued, i.e.  $\arg z = \theta + 2\pi k, k \in \mathbb{Z}$ .
- The **principal argument**,  $\text{Arg } z$ , is such that  $-\pi < \text{Arg } z \leq \pi$ .
- d) What is  $\bar{z}$ ?
- **Recall:** If  $z = a + bi$ , then the **complex conjugate** of  $z$  is:  $\bar{z} = a - bi$ .



## Examples:

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- 4. Express  $(\sqrt{3} - i)^6$  in polar form.



## Examples:

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- 5. Find the solutions to the equation  $z^3 = -1$ .
- Recall:  $z^{\frac{1}{n}} = \sqrt[n]{r}[\cos(\frac{\theta}{n} + \frac{2k\pi}{n}) + i\sin(\frac{\theta}{n} + \frac{2k\pi}{n})]$ ,  $k = 0, 1, \dots, n - 1$ .





## **Examples:**

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- **6. a)** Find the square roots of  $2i$ .
- **b)** Express your two roots in rectangular coordinates.

