Math 1B03/1ZC3 - Tutorial 4

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Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
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Examples:

- **1.** Consider
$$A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}.$$
- **a)** What are the eigenvalues of A?.
- **Recall:** If A is square, then $\boldsymbol{x} \in \mathbb{R}^n$ such that $\boldsymbol{x} \neq 0$ is called an **eigenvector** of A if $A\boldsymbol{x} = \lambda\boldsymbol{x}$ for some $\lambda \in \mathbb{R}$. (i.e. $A\boldsymbol{x}$ is a scalar multiple of $\boldsymbol{x}$).
- The scalar λ is called an **eigenvalue** of A, and $\boldsymbol{x}$ is λ's corresponding eigenvector.
- $A\boldsymbol{x} = \lambda\boldsymbol{x} \Leftrightarrow A\boldsymbol{x} - \lambda\boldsymbol{x} = 0 \Leftrightarrow (A - \lambda I)\boldsymbol{x} = 0$.
- We know $\det(A) = 0 \Leftrightarrow A$ <u>not</u> invertible$\Leftrightarrow A\boldsymbol{x} = 0$ has non-trivial solutions.
- So, since we're looking for vectors $\boldsymbol{x}$ such that $(A - \lambda I)\boldsymbol{x} = 0$ and we know that $\boldsymbol{x} \neq 0$ by definition, then by our equivalent statements about inverses that must mean that $\det(A - \lambda I) = 0$.
- So, λ is an **eigenvalue** of $A \Leftrightarrow$ it satisfies the equation $\det(A - \lambda I) = 0$.

Examples:

- **b)** Find all eigenvectors of A.

Examples:

- **2.a)** Find all eigenvalues of

$$A = \begin{pmatrix} 3 & 6 & -6 \\ -1 & -4 & 5 \\ 2 & 2 & -1 \end{pmatrix}.$$

- **b)** Find all eigenvectors corresponding to $\lambda = -3$.
- **c)** Is A invertible?

Examples:

- **3.a)** Find the eigenvalues of

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

- **b)** Find all eigenvectors corresponding to $\lambda = 2$.

Examples:

- **4.)** Consider

$$A = \begin{pmatrix} 5 & -3 \\ a & b \end{pmatrix}$$

and suppose

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is an eigenvector of A. What must the eigenvalue λ corresponding to $\boldsymbol{x}$ be?

Examples:

- **5.)** Find all eigenvalues and eigenvectors of A^{10}, if

$$A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}.$$