Math 1B03/1ZC3 - Tutorial 4

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Tutorial Info:

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Examples:

- **1.** Consider
$$A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 2 \\ 1 & 5 & 3 \end{pmatrix}.$$
- **Recall:** For a 2×2 matrix
$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$
$\det(B) = ad - bc$.
- **Recall:** If D is a square, then the **minor of entry $\mathbf{a_{ij}}$**, $\mathbf{M_{ij}}$ is the determinant of the submatrix that remains after the i^{th} row and j^{th} column are deleted from D.
- **Cofactor of entry $\mathbf{a_{ij}}$, $\mathbf{C_{ij}}$:** is kM_{ij}, where $k = 1$ or -1 in accordance with the pattern in the checkerboard array:
$$B = \begin{pmatrix} + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix},$$

Examples:

- **a)** Find M_{11}, M_{12}, M_{13}, C_{11}, C_{12}, and C_{13}
- **b)** Find $\det(A)$.
- **Recall:** You can find $\det(A)$ by multiplying the entries in any row or column by their corresponding cofactor and adding the resulting products.
- **Note:** We could have chosen a different row or column.

Examples:

- **2.** Consider
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}.$$
Find $\det(A)$.
- **Note:** Choosing a row or column with lots of zeros makes things easier!

Examples:

- **2.** Consider

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}.$$

- **a** Find $\det(A)$.
- **Recall:** How do elementary row operations affect matrices?
- Let B be a square matrix, and let C denote what B becomes after each row operation.
  1. Multiply row by nonzero scalar k: $\det(C) = k\det(B)$.
  2. Switch any 2 rows: $\det(C) = -\det(B)$.
  3. Add a multiple of a row to an existing row: $\det(C) = \det(B)$.
- Doing the same operations on B's columns yield the same results.

Examples:

- **b)** Consider

$$B = \begin{pmatrix} t & 2t & 3t \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix},$$

for $t \in \mathbb{R}$. Find $\det(B)$.

Examples:

- **4.** Suppose $\det(A) = 3$, $\det(B) = 9$, $\det(C) = 2$. What is $\det(X)$, if $BX = 6C^T A$.
- **Recall:** We know the following properties concerning determinants:
  (a) $\det(A) = \det(A^T)$
  (b) $\det(AB) = \det(A)\det(B)$
  (c) $\det(kA) = k^n \det(A)$, where $k \in \mathbb{R}$, and A is a $n \times n$ matrix.
  (d) $\det(A) \neq 0 \Leftrightarrow A$ is invertible.
  (e) $\det(A^{-1}) = \frac{1}{\det(A)}$.

Examples:

- **5.a)** Consider

$$A = \begin{pmatrix} 1 & x & 2 \\ 3 & 1 & -1 \\ -1 & 2 & 2 \end{pmatrix}.$$

  When is A singular?

- **Recall:** A matrix A is called **singular** if A is *not* invertible.
- Also, we know that A invertible $\Leftrightarrow \det(A) \neq 0$, so A singular $\Leftrightarrow \det(A) = 0$.
- So, we're looking for the values of x such that $\det(A) = 0$.
- **b)** When is A invertible?

Examples:

- **6.** Consider

$$A = \begin{pmatrix} 0 & 2 & 1 \\ -1 & -3 & 1 \\ -2 & -1 & -2 \end{pmatrix}.$$

- Find A^{-1} using the adjoint method.
- **Recall:** If A is invertible, then $A^{-1} = \frac{1}{\det(A)} adj(A)$, where

$$adj(A) = \begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{pmatrix}^T.$$

Examples:

- **7.** Solve the following linear system using Cramer's Rule: $3x+2y=1$, $5x+4y=-1$.
- **Cramer's Rule:** If $Ax=b$ is a system of n linear equations in n unknowns such that $\det(A)\neq 0$, then $Ax=b$ has a unique solution.
  This solutions is: $x_1=\frac{\det(A_1)}{\det(A)}$, $x_2=\frac{\det(A_2)}{\det(A)}$, $\ldots$, $x_n=\frac{\det(A_n)}{\det(A)}$, where A_j is the matrix obtained by replacing the entries in the jth column of A by the entries in the matrix b.