Math 1B03/1ZC3 - Tutorial 2

Jan. 21st/24th, 2014

Tutorial Info:

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Does the Commutative Law for Multiplication hold for Matrices?, i.e. is it always true that $AB = BA$?

- Well, we know that that if A and B are not the same size, then BA may not even be defined.
- **e.g.** If

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 0 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 10 & 2 & 0 & -4 \\ 7 & 2 & 1 & -3 \\ 9 & 6 & 7 & -5 \end{pmatrix},$$

but BA is not defined.

- So no, it is not true in general that $AB = BA$.

Does the Commutative Law for Multiplication hold for Matrices?

- What if A and B are both square (i.e. A and B are both $n \times n$ matrices)?
- Does $AB = BA$ for any possible A and B?
- Can you think of a counterexample?

Does the Commutative Law for Multiplication hold for Matrices?

- Is it ever possible to find an A and B such that $AB = BA$?

Zero Divisors?

- For real numbers, we know that $ab = 0 \Rightarrow a = 0$ or $b = 0$.
- Is this true for matrices? (i.e. if we have two matrices A and B such that $AB = 0$, is it true that we must have $A = 0$ or $B = 0$?)

Cancellation Law?

- For real numbers, we know that $ab = ac \Rightarrow b = c$.
- Does this hold true in general for matrices? (i.e. $AB = AC \Rightarrow B = C$?

Recap: In general, it is not true that:

- $AB = BA$ (i.e. multiplicative commutativity fails)
- $AB = 0 \Rightarrow A = 0$ or $B = 0$ (i.e. $\exists$ non-zero zero divisors)
- $AB = AC \Rightarrow B = C$ (i.e. cancellation law fails)

Multiplicative Identity

- In $\mathbb{R}$ we have the number 1, and we know $a \times 1 = 1 \times a = a$.
- For matrices, this "1" is known as the *identity matrix*, e.g. if A is $m \times n$, then $A \times I_{n \times n} = A$.
- **e.g.**

Multiplicative Inverse

- In $\mathbb{R}$ we know that for every a such that $a \neq 0$ there exists a^{-1} such that $aa^{-1} = a^{-1}a = 1$. **e.g** $2 \times \frac{1}{2} = 1 = \frac{1}{2} \times 2$.
- If A is a square $(n \times n)$ matrix such that $\exists$ a B such that $AB = I_{n \times n} = BA$, then A is said to be **invertible**, (a.k.a **nonsingular**), and B is called the inverse of A, $(B = A^{-1})$.
- If A is a 2×2 matrix, then

$$A^{-1} = \frac{1}{ad - bc} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

b/c:

For 2x2 Matrices:

- det(A)$= ad - bc$.
- A is nonsingular $\Leftrightarrow$ det(A)$\neq 0$.
- So, det(A)$= 0 \Leftrightarrow A$ is singular (i.e. A is not invertible).

Examples:

- **1.** Let

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}.$$

- **a)** Is A invertible?.
- **b)** Find A^{-1}.
- **c)** Is B invertible?.
- **d)** Find B^{-1}.
- **e)** Find $(AB)^{-1}$.

Examples:

- **2.** Let

$$A = \begin{pmatrix} 4 & x \\ x & 1 \end{pmatrix}.$$

For what values of x is A singular?

Examples:

- **3.** Solve for X: $A(X+B) = CA$ (where A is invertible).

Examples:

- **4.** Solve for X: $(2E+F)^T = G^{-1}X^T + F^T$.

Examples:

- **5.** Find the inverse of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 5 \\ 3 & 2 & 3 \end{pmatrix}$$

using row operations.

Examples:

- **6. a)** Solve for W: $2EWF^2 = (E^T F)^2$.
- **b)** What sizes must F and W be in order for W to have a unique solution if E is $3 \times n$?