

Math 1B03/1ZC3 - Tutorial 11



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Tutorial Info:

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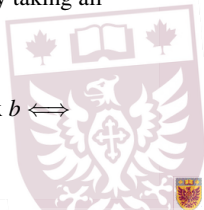
Examples:

- 1. Consider the following sets of vectors:

$$S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\},$$

$$S_3 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

- a) Which sets span \mathbb{R}^3 ?
- Recall:** The **span** of a set $S = \{w_1, \dots, w_r\}$, is the subspace formed by taking all possible linear combinations of the vectors in S . **i.e.**
 $\text{span}(S) = \{\alpha_1 w_1 + \dots + \alpha_r w_r \mid \alpha_1, \dots, \alpha_r \in \mathbb{R}\}$.
- Recall:** If A is square, then $Ax = b$ is consistent for every $n \times 1$ matrix $b \iff \det(A) \neq 0$.



Examples:

- b) Is the vector

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ in the span of } S_1? S_2? S_3?$$

- c) Which of these sets are linearly independent?
- **Recall:** If a set of vectors $S = \{v_1, \dots, v_r\}$ is such that the equation $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_r v_r = \vec{0}$ has only the trivial solution (i.e. $\alpha_1 = \dots = \alpha_r = 0$), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.



Examples:

- 2. Which of the following form a basis for \mathbb{R}^2 ?
- **Recall:** A set $S = \{v_1, \dots, v_n\}$ of vectors, where $v_1, \dots, v_n \in V$ is called a **basis** for V if:
 1. The vectors in S are linearly independent.
 2. S spans V .
- $S = \{(1, 0), (0, 1)\}$ is called the **standard basis** for \mathbb{R}^2 .
- If $\{v_1, \dots, v_n\}$ is a basis for V then:
 1. If a set S of vectors from V has $> n$ vectors, then S is linearly dependent.
 2. If S has $< n$ vectors, then S does not span V .

■ a)

$$S := \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}.$$

■ b)

$$T := \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$



Examples:

- 3. We know

$$T := \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

forms a basis for \mathbb{R}^2 .

- a) Find the coordinate vector of $v = (3, 5)$ relative to the basis T . i.e. Find $[v]_T$.
- **Recall:** If $S = \{v_1, \dots, v_n\}$ is a basis for V , and $w = k_1 v_1 + k_2 v_2 + \dots + k_n v_n$ for $k_1, \dots, k_n \in \mathbb{R}$, then $[w]_S = (k_1, \dots, k_n)$ is called the **coordinate vector of v relative to S** .
- b) Find the vector $w \in \mathbb{R}^2$ whose coordinate vector relative to T is $[w]_T = (4, 2)$.



Examples:

- 4.) Which of the following are a basis for P_2 (where P_2 is the vector space of all polynomials of degree ≤ 2 ; i.e. $P_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbb{R}\}$).
- $W = \{x^2, x + 1, x^2 + x + 1\}$
- $X = \{x, 1, 0\}$
- $T = \{x^2 + x + 1, x^2 + x, x + 1\}$
- $Y = \{x^2, x, 1, 0\}$

