Math 1B03/1ZC3 - Tutorial 11

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Tutorial Info:

- **Website:** http://ms.mcmaster.ca/∼dedieula.
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Examples:

- **1.** Consider the following sets of vectors:

$$S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\},$$

$$S_3 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

- **a)** Which sets span $\mathbb{R}^3$?

- **Recall:** The **span** of a set $S = \{w_1, \ldots, w_r\}$, is the subspace formed by taking all possible linear combinations of the vectors in S. **i.e.** $\text{span}(S) = \{\alpha_1 w_1 + \ldots \alpha_r w_r | \ \alpha_1, \ldots, \alpha_r \in \mathbb{R}\}$.

- **Recall:** If A is square, then $Ax = b$ is consistent for every $n \times 1$ matrix $b \Longleftrightarrow$ $\det(A) \neq 0$.

Examples:

- **b)** Is the vector

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ in the span of } S_1?\ S_2?\ S_3?$$

- **c)** Which of these sets are linearly independent?

- **Recall:** If a set of vectors $S = \{v_1, \ldots, v_r\}$ is such that the equation $\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_r v_r = \bar{0}$ has only the trivial solution (i.e. $\alpha_1 = \ldots = \alpha_r = 0$), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.

Examples:

- **2.**Which of the following form a basis for $\mathbb{R}^2$?
- **Recall:** A set $S = \{v_1, \ldots, v_n\}$ of vectors, where $v_1, \ldots, v_n \in V$ is called a **basis** for V if:
  1. The vectors in S are linearly independent.
  2. S spans V.
- $S = \{(1,0),(0,1)\}$ is called the **standard basis** for $\mathbb{R}^2$.
- If $\{v_1, \ldots, v_n\}$ is a basis for V then:
  1. If a set S of vectors from V has $> n$ vectors, then S is linearly dependent.
  2. If S has $< n$ vectors, then S does not span V.
- **a)**

$$S := \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}.$$

- **b)**

$$T := \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

Examples:

- **3.** We know
$$T := \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$
forms a basis for $\mathbb{R}^2$.
- **a)** Find the coordinate vector of $v = (3,5)$ relative to the basis T. i.e. Find $[v]_T$.
- **Recall:** If $S = \{v_1, \ldots, v_n\}$ is a basis for V, and $w = k_1 v_1 + k_2 v_2 + \ldots + k_n v_n$ for $k_1, \ldots, k_n \in \mathbb{R}$, then $[w]_S = (k_1, \ldots, k_n)$ is called the **coordinate vector of** v **relative to** S.
- **b)** Find the vector $w \in \mathbb{R}^2$ whose coordinate vector relative to T is $[w]_T = (4,2)$.

Examples:

- **4.)** Which of the following are a basis for P_2 (where P_2 is the vector space of all polynomials of degree ≤ 2; i.e. $P_2 = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$.
- W=$\{x^2, x+1, x^2+x+1\}$
- X=$\{x, 1, 0\}$
- T=$\{x^2+x+1, x^2+x, x+1\}$
- Y=$\{x^2, x, 1, 0\}$